Designing a Product Distribution Network Using Mixed Integer Planning

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Abstract

In order to achieve the commercial goals and to be successful in the distribution management challenges, companies have to pay special attention to the distribution network. The effective and successful management of the distribution network results in meeting the customers' needs directly or indirectly. In this research, customers include retailers and wholesalers. In order to approach to the retailers in a region, a warehouse at that region is considered, in a way that the customers provide their needs from this warehouse in a short time. In this research, a network has been designed which considers the needs of customers & producers. Factors considered include profit, lead time, product variety, power of retailer and credit of retailer. Considering these factors and retailers' and wholesalers' supplying-manners, the best distribution options for each customer determined in this research. In order to determine the best distribution network the mixed integer planning model is used. This model determined the optimal percent of distribution from the regional warehouse & the producer's warehouse (from 6%/94% to 24%/76%), and finally states the profit resulted from establishing a regional warehouse for the producer (approximately a monthly increase of 3300 million rials in profit of producer).

Keywords: Supply chain, Distribution network, Multi-objective optimization, AHP, Sensitivity analyze

1. Introduction

In the current age, in order to survive in the competition area, it is necessary to present various products with high quality and reasonable price, along with fast services proportional to the customer's demand. This research studies the issues and problems related to the distribution network in a region and tried to handle the different products to the retailers & wholesalers in a short time and by a proper profit. Also it tries to select those customers who are wealthy so that the company can guarantee its survival by creating long-time relations with the desired customers. Creating a long-time & continuous relationship results in establishing mutual trust between suppliers and customers, and, along this relationship, useful information may be shared along the supply chain, in a way that any section of the chain can receive its share from the chain profit.

In order to design the distribution network properly, the customers' & producers' interests & benefits should be considered totally. This has been considered in designing this model. The recommended distribution is in a way that maximizes the profit of the customers set. This research, not only studies the cost-reducing strategies throughout the goods-presenting canal, but also creates balance between the customers' increasing demand and effective & on-time services. Second section studies the research literature and explains the models propounded in the distribution section. Third section describes the distribution network designing model and determines the methods of supplying the customers. In a way that the best network for distributing is determined. Forth section explains Barez Co. which is under study; it also describes the structure of the

current distribution network of Barez Co. and describes its deficiencies in detail. In the final section the research results & findings are determined, the best choice for each customer is recommended regarding the factors of distribution network model and the appropriate proportion of distribution by the producer is propounded.

2. Research literature

Designing a distribution network can be so wide and extended. Many decisions may be made while designing the supply chain. Some researchers have carried out their research on the base of number of warehouses required in any region. Some researches are related to the products level and warehouse stock, while other try to optimize the sending programs. Yet some researches study the specialized applications. In most of the researches, determining the factories and warehouses required in a region is considered while designing the distribution network.

Bilgen (2010) considers the problems of the production & distribution program and presents a model in which products with more profit are produced in the factory lines and are handled to the distribution centers. This phase-model is applied in order to optimize and integrate the production & distribution program. The application and flexibility of this model has been shown by a case under study at the consumed-goods industry.

Jonsson, Kjellsdotter & Rudberg (2007), using APS advanced software, propounded a programming system for problem-solving in the strategic and tactical level, and studied this software on three cases under study. This software reduced the expenses in one case, and optimized the handling services, production program and efficiency in the other two. APS is a standards, supporting, decision-making system which has the ability to be applied in the complicated and various programs. Certainly, for using this software, so much information shall be gathered.

Wang, Sun & Yang (2005) proceeded to some problems related to decision-making about the facilities location, stock & transportation, and formulated a multi-step distribution network design. This model may be applied in the tactical & operational decisions. Finally, they stated that in the competition market, customer-service level is promoted and thus the customer-service expense is increased.

Lee, Kang & Lee (2008) have studied the distribution programming model for some levels of the supply chain network. Goal function minimizes the logistic expenses. Logistic expenses include the expenses related to loading, stock maintaining and transportation.

Sourirajan, Ozsen & Uzsoy (2008) proceeded to a two-step supply chain. They tried to meet the retailers' needs considering the production facilities. The goal of this research is determining the distribution centers in the network, and number and location of the facilities, and also minimizing the stock of pipe stock and reliable stock expenses. This model establishes a relationship between circulation in the network, sending time, and reliable stock level. The genetics algorithm is used for solving this model.

June Young et al (2010) study the management of reliable stock in the supply chain. In the medicine, chemistry and petrochemistry industries, the supply chain consists of the multi-step facilities of production, distribution centers/ warehouse, minor supply networks and the end customer. The operation of the supply chain in confronting the different markets is usually evaluated by the services level. Reliable stock is introduced as an important obstacle against not-assurance of the customer's order in the supply chain. More reliable stock for higher customer-service level increases the operational expenses of the supply chain, thus, the proper and optimal service level shall be studied. In this research a linear programming model is introduced considering the reliable stock management, and the application of this model is indicated in a case under study.

Li, Chu & Chen (2011) have proceeded to the problems of the stock route in the three level of the distribution system, which are a seller, a warehouse, and some retailers that are distributed in different locations. Each

retailer is faced with a distinct & clear demand of any product. The retailer's demand is met through these two ways: by the seller from the warehouse, Directly by the seller. Stock is stored either in the warehouse or by the retailer. The objective of this research is determining a strategy composed of transportation and stock which reduces the transportation expenses in long time and meets the retailers' needs completely. In this research the retailers are divided to different groups and needs of any group are provided through a different route.

Ambrosino & Scutella (2005) have carried out a research on the distribution network. In their research, company has one warehouse through which the different goods are transported rapidly, and the customers are divided regarding the different region. They have studied on some parameters which minimize the distribution expenses, such as finding the accurate location of the warehouses, determining the number of vehicles traveling in any region, determining the routes of vehicles in any region, starting, terminating or developing the activity of warehouses.

Tsiakis & Papageorgiu (2008) introduced financial limits (production expenses, transportation expenses & commitment of material flow inside the network), and prioritized the operational limits (quality, supply and production limits, products allocating and the work-load balance). They suggest MILP model for optimizing the production-distribution system, and considering the financial and operational limits.

Aghezzaf, Raa & Landeghem (2006) have studied the stock problems. They determined the distribution programs and minimize the operational expenses, storing expenses and stock average; in a way that there is no extra stock at the sale point along the programming horizon.

Ahire, Gorman, Dwiggins & Mudry (2007) use the techniques of the operational research in which some standards are registered in the company which allocate the products optimally and minimize the expenses set of the distribution centers through the production-distribution network. They use regression, optimizing models and stimulating models for problem-solving.

3. Model of distribution network design

Due to lack of relationship between producer and retailers, in this model, it is tried to increase the relationship between retailer and producer and to increase the profit of producer by decreasing selling to wholesalers and increasing selling to retailers. In this study, Model of distribution network of Cintron & Ravindran & Ventura (2010) has been used.

In this study, customers are retailers and wholesalers. The customers may supply their needs in several manners. Wholesalers may meet their needs from the producer or distributional centers. Retailers meet their needs from wholesalers, producer and distributional center.

As a result, there are four options for the customer in order to meet their needs as follows:

1-Producer…..regional distribution center….retailer

2-Producer…..retailer

3-Producer….regional distribution center… wholesaler…. Retailer

4-Producer…wholesaler…retailer

There are a number of suppliers, wholesalers and retailers in this study. P= $\{1,2,\ldots,n_p\}$ shows a set of producer, R=$\{1,2,..,n_r\}$ a set of retailer and W= $\{1,2,..,n_w\}$ a set of wholesalers.

The first option shows a situation when factory P dispatches the order of regional distribution center DC and channel of regional distribution DC, dispatches it to the retailer R. in the second option, the order is dispatched directly from supplier P to the retailer R. in the third option, factory P dispatches the order of regional distribution center DC. Distribution center DC sends the order of the wholesaler W and the wholesaler sends the order of retailer R. in the fourth option, the producer P sends the order of the wholesaler W and the wholesaler also sends the order of the retailer R. finally the K set that involves all kinds of vehicles is used for supplying demands of customers. Each vehicle has its own cost and capacity. In this

study, customers mean retailers and wholesalers. In next section, factors, limitations and object function will be discussed.

3.1 Customer satisfy factor

3.1.1 Profit factor

One of variables in this model is to maximize the profit. The profit is selling margin minus distribution costs.

*Selling margin is selling price minus buying price. The product will be delivered to the retailers in three ways: by the producer, distribution center. The wholesaler also will be supplied by the producer and distribution centers. Each sector that supplies the customers will represent a price for selling their products. So selling margin of each of these options is different. The customers (retailers and wholesalers) tend to meet their needs from the producer because the buying price from the producer is lower than distributional centers.

*Distribution costs involve transportation costs, warehousing costs, insurance, and servicing costs to the customer. Transportation cost is a function of demand amount, type of vehicle and the distance from the source to the destination. Transportation efficiency depends on the time, equipments, the stock and personnel. Planning for transportation vehicles, classification and their correct application play important roles in using optimally from these vehicles. Transportation costs are a function of the type of vehicle (dependent on customer demand) and the distance between the customer and distribution centers. In this research, it is assumed that the orders received from the customers are not the same. Warehousing costs depend on the type of the warehouse. If the warehouse is rented, rent costs, personnel wage and warehouse insurance will be fixed costs. Discharge and distribution costs are variable costs. If the warehouse is a general or public one, warehouse costs involve insurance (fixed cost) and warehousing cost (variable cost). A public warehouse with indefinite capacity is considered in this study. Considering the costs mentioned in distribution channel, if demand of the customer is one truck, it will be carried directly from the supplier. When demand of the customer is limited, the retailers want to supply their needs from distribution centers and wholesalers.

3.1.2 Factor of lead time

Lead time is the interval between issue of the order and receiving order by the customer. This time is different regarding different distributional options. Lead time relates to direct dispatch from the supplier. By creating more facilities near the customer, the lead time can be reduced. The lead time will be obtained by opinion sampling from wholesalers and retailers. So lead time from the producer, distribution center and wholesalers will be consulted by the customers and the average of these numbers will be considered as the lead time for every producer.

3.1.3 Factor of product variety

With more product variety of suppliers, the order of customers will be delivered with low price. Good variety is high in warehouse of the producer and he can deliver a high level of good varieties to the customers.

Variety is the same for wholesalers and regional distribution centers. Product variety of producer is calculated as follows: the number of product varieties at warehouse of producer during one month. Product variety in distribution centers and wholesaler will be calculated similarly.

3.2 Important factors for producer

* Power of retailer

The more the power of the retailer, the more tendencies the producer has to continue cooperation. Continuous purchase of the retailer will increase the power to pay the bills timely so the financial status of the customer will increase suitably. Its number is chosen from 1 to 10.

* Credit of the retailer

Performance of the retailer in the past, his fame and generosity consist customer credit. Good background increase credit of the retailer. The number is chosen between 1 and 10.

3.3 Notation

Model parameters

M very large number
rd_{rp} monthly demand of retailer r from producer p (rings)
rdw_{rp} monthly demand of retailer r from wholesaler w (rings)
wd_{wp} monthly demand of wholesaler w from producer p (rings)
ra_{rp} monthly demand of retailer r from DC (rings)
rb_{wp} monthly demand of wholesaler w from DC (rings)
MD minimum monthly demand to order direct shipment (rings)
vw product variety rating from DC to customer
vx_p product variety rating from producer p to customer
vb product variety rating from wholesaler to retailer
vc_k capacity of vehicle type k (rings)
dclt lead time from the DC to a customer
slt_p lead time from producer p to a customer
dlt lead time from wholesaler to retailer
cp_r credit performance rating for retailer r
c_r power rating for retailer r
tca_{rp} monthly transportation cost for delivering the demand of retailer r from DC
tcb_{wp} monthly transportation cost for delivering the demand of wholesaler w from DC
tcw_{rp} monthly transportation cost for delivering the demand of retailer r from wholesaler w
tcc_{rp} monthly transportation cost for delivering the demand of retailer r from producer p
tcd_{wp} monthly transportation cost for delivering the demand of wholesaler w from producer p
vcm_{rp} monthly Selling margin of retailer r for the demand from producer p
vcc_{wp} monthly Selling margin of wholesaler w for the demand from producer p
vcw_{rp} monthly Selling margin of retailer r for the demand from wholesaler w
vca_{rp} monthly Selling margin of retailer r for the demand from DC
vcb_{wp} monthly Selling margin of wholesaler w for the demand from DC
aba_{rp} monthly inventory holding cost per ring of the demand of retailer r from DC
abb_{wp} monthly inventory holding cost per ring of the demand of wholesaler w from DC
abm_{rp} monthly inventory holding cost per ring of the demand of retailer r from producer p
abc_{wp} monthly inventory holding cost per ring of the demand of wholesaler w from producer p
abw_{rp} monthly inventory holding cost per ring of the demand of retailer r from wholesaler w
MP monthly Ideal profit used for the profit goal
MLT monthly lead time used for the lead time goal
MW monthly power value used for the power goal
MCP monthly credit performance value used for the credit goal
MV monthly product variety value for the product variety goal

Model variables

y_{pr} 1 if option 1 is selected for retailer r when receiving from plant p, and 0 otherwise
t_{pr} 1 if option 2 is selected for retailer r when receiving from plant p, and 0 otherwise
x_{pwr} 1 if option 3 is selected for retailer r when receiving from plant p via wholesaler w, and 0 therwise
q_{pwr} 1 if option 4 is selected for retailer r when receiving from plant p via wholesaler w, and 0 otherwise
a_r 1 if retailer r receives product from the DC or the plants, and 0 if it receives it from wholesaler
b_{pw} 1 if wholesaler w is supplied directly from plant p, and 0 if it is supplied from the DC
g_{wr} 1 if wholesaler w supplies retailer r, and 0 otherwise

nk_{kr} number of vehicle type k (or number of trips) needed to supply the monthly demand of retailer r from the DC

3.4 Model constraints

There is only one option per purchase for every customer (retailer or wholesaler). The retailer can choose three options including wholesaler, distribution center and producer. The wholesaler can also choose only one option between distribution center and producer. This limitation is as follows:

$$y_{pr} + t_{pr} + \sum_{w \in W} x_{pwr} + \sum_{w \in W} q_{pwr} = 1 \quad p \in P, r \in R \qquad (1)$$

Another limitation is that the retailer can receive his order from producer, distribution center of the producer or the wholesaler. The retailer has only one choice per purchase and can choose one of above mentioned options. So if option 1 or 2 is chosen for every producer, options 3 or 4 will not be chosen. The set of these limitations have been shown in equations 2 and 3.

$$\sum_{p \in P} y_{pr} + \sum_{p \in P} t_{pr} \leq Ma_r \qquad r \in R \qquad (۲)$$

$$\sum_{p \in P} \sum_{w \in W} x_{pwr} + \sum_{p \in P} \sum_{w \in W} q_{pwr} \leq M(۱ - a_r) \qquad r \in R \qquad (۳)$$

a_r is a variable of one and zero and the retailer, r, can only choose limit 2 or 3 regarding limitation factor. M is a large number in these two equations so that it causes that only one of these equations is chosen. A wholesaler can meet his needs by regional distribution centers or directly from the producer. But it is possible that the wholesaler chooses options 3 or 4. Equations 4 and 5 show this limitation for the wholesaler. In this equation, variable of one and zero, b_{pw}, is introduced. When b_{pw} is equal to one, the wholesaler can be supplied directly from the supplier and if it is zero, the wholesaler will be supplied by regional distribution center.

$$\sum_{r \in R} x_{pwr} \leq M(۱ - b_{pw}) \qquad p \in P, w \in W \qquad (٤)$$

$$\sum_{r \in R} q_{pwr} \leq Mb_{pw} \qquad p \in P, w \in w \qquad (٥)$$

Another limitation in this model is that the retailer in each order can buy from a wholesaler and only one wholesaler is chosen by the retailer. Variable of one and zero, g_{wr}, is introduced for it. If g_{wr} is equal to one, it means that the retailer r is supplied by the wholesaler w and if it is zero, he will be supplied by producer or distribution center. The set of these limitations has been shown in equations 6 and 7.

$$\left(\sum_{p \in P} x_{pwr} + \sum_{p \in P} q_{pwr} \right) \leq Mg_{wr} \qquad w \in W, r \in R \qquad (٦)$$

$$\sum_{w \in W} g_{wr} \leq ۱ \ , \quad r \in R \qquad (٧)$$

So the wholesaler or the retailer should have a big demand in order to supply their needs (a truck with complete capacity). The demand may include one or more products. Equations 8 and 9 show these

limitations. In this equation, rd_{rp} shows demand of the retailer r from producer p, and wd_{pw} shows demand of wholesaler w from producer p.

$$rd_{pr} \geq MDt_{pr} \ , \quad p \in P, \quad r \in R \qquad (۸)$$

$$wd_{pw} + \sum_{r \in R} rd_{pr} q_{pwr} \geq MDb_{pw} \quad p \in P, r \in R \qquad (۹)$$

The lowest capacity of transportation of plant is equal to 550 rings. In fact MD is equal to 550 rings. When retailers and wholesalers have demands less than 550 rings, they supply their needs from distribution centers. Transportation costs depend on the type of vehicle. Vehicles have different cost. So transportation costs in distribution centers should be determined by an equation. Equations 10 and 11 show a set of limitations that choose vehicle. These vehicles are used for supplying demands of retailers and wholesalers.

$$\sum_{p \in P} rd_{pr} y_{pr} \leq \sum_{k \in K} nk_{kr} \ vc_k \qquad (۱۰)$$

$$\sum_{p \in P} wd_{pw}(۱ - b_{pw}) + \sum_{p \in P} \sum_{r \in R} rd_{pr} x_{pwr} \leq \sum_{k \in K} bh_{kw} vc_k \ , \quad w \in W \qquad (۱۱)$$

3-5 Model objectives

Several objects have been considered in this model.

* Maximize profit customer (wholesaler and retailer)

$$\begin{aligned} &\sum_{p \in P} \sum_{r \in R} vca_{pr} y_{pr} + \sum_{p \in P} \sum_{r \in R} vcm_{pr} t_{pr} + \sum_{p \in P} (\sum_{w \in w} vcb_{pw}(۱ - b_{pw}) + \sum_{r \in R} vcw_{pr} x_{pwr}) + \sum_{p \in P} (\sum_{w \in W} vcc_{pw} b_{pw} \\ &+ \sum_{r \in R} vcw_{pr} q_{pwr}) \\ &- \sum_{p \in P} \sum_{r \in R} tca_{pr} y_{pr} - \sum_{p \in P} \sum_{r \in R} tcc_{pr} t_{pr} - \sum_{p \in P} (\sum_{w \in W} tcb_{wp}(۱ - b_{pw}) \\ &+ \sum_{r \in R} tcw_{pr} x_{pwr}) - \sum_{p \in P} (\sum_{w \in W} tcd_{pw} b_{pw} + \sum_{r \in R} tcw_{pr} q_{pwr}) - \sum_{p \in P} \sum_{r \in R} aba_{pr} y_{pr-} \sum_{p \in P} \sum_{r \in R} abm_{pr} t_{pr} \\ &- \sum_{p \in P} (\sum_{w \in W} abb_{pw}(۱ - b_{pw}) + \sum_{r \in R} abw_{pr} x_{pwr}) - \sum_{p \in P} (\sum_{w \in W} abc_{pw} b_{pw} + \sum_{r \in R} abw_{pr} q_{pwr}) \end{aligned} \qquad (12)$$

* Minimize lead time

$$dclt\left(\sum_{p \in P} \sum_{r \in R} y_{pr} + \sum_{p \in P} \sum_{r \in R} slt_p \ t_{pr}\right) + dlt(\sum_{p \in P} \sum_{w \in W} \sum_{r \in R} x_{pwr} + \sum_{p \in P} \sum_{w \in W} \sum_{r \in R} q_{pwr}) \qquad (13)$$

* Maximize power of retailer

$$\sum_{r \in R} c_r(\sum_{p \in P} y_{pr} + \sum_{p \in P} t_{pr}) + \sum_{r \in R} c_r(\sum_{p \in P} \sum_{w \in W} x_{pwr} + \sum_{p \in P} \sum_{w \in W} q_{pwr}) \qquad (14)$$

C_r: evaluation of retailer (from 1 to 10) based on power of the retailer growth or relation of the retailer with the company. This assessment shows tendency of the producer for protection of the retailer. Low score to the retailer shows that the producer wants to cooperate with the wholesaler.

* maximize credit of retailer

$$\sum_{r\epsilon R} cp_r\left(\sum_{p\epsilon P} y_{pr} + \sum_{p\epsilon P} t_{pr}\right) + \sum_{r\epsilon R} cp_r\left(\sum_{p\epsilon P}\sum_{w\epsilon W} x_{pwr} + \sum_{p\epsilon P}\sum_{w\epsilon W} q_{pwr}\right) \qquad (15)$$

CP_r: shows assessment of retailers' credit and it is between 1 and 10. High score shows that the producer wants to keep the retailer and low score to the retailer shows that the producer wants to cooperate with the wholesaler.

* maximize product variety

$$\mathbf{vw}\sum_{p\epsilon P}\sum_{r\epsilon R} y_{pr} + \sum_{p\epsilon P}\sum_{r\epsilon R} vx_p\, t_{pr} + \mathbf{vb}\left(\sum_{p\epsilon P}\sum_{w\epsilon W}\sum_{r\epsilon R} x_{pwr} + \sum_{p\epsilon P}\sum_{w\epsilon W}\sum_{r\epsilon R} q_{pwr}\right) \qquad (16)$$

3-6 Goal programming

The method of goal programming is used to solve problems that have more than one object function. Goal programming is very similar to linear planning except that its object function is more than one object. In this study, five ideals of profit, lead time of lead time, power of retailer, credit of retailer and product variety are considered. Above mentioned objects will be solved separately regarding the model limitations. Regarding the limitations of that month, each object is considered per month. This value is called as ideal value of that factor.

* Profit goal

$$\begin{aligned}
&\sum_{p\epsilon P}\sum_{r\epsilon R} vca_{pr} y_{pr} + \sum_{p\epsilon P}\sum_{r\epsilon R} vcm_{pr} t_{pr} + \sum_{p\epsilon P}\left(\sum_{w\epsilon w} vcb_{pw}(1 - b_{pw})\right.\\
&\quad \left. + \sum_{r\epsilon R} vcw_{pr} x_{pwr}\right) + \sum_{p\epsilon P}\left(\sum_{w\epsilon W} vcc_{pw}\, b_{pw}\right.\\
&\quad \left. + \sum_{r\epsilon R} vcw_{pr}\, q_{pwr}\right)\\
&\quad - \sum_{p\epsilon P}\sum_{r\epsilon R} tca_{pr} y_{pr} - \sum_{p\epsilon P}\sum_{r\epsilon R} tcc_{pr}\, t_{pr} - \sum_{p\epsilon P}\left(\sum_{w\epsilon W} tcb_{wp}(1 - b_{pw})\right.\\
&\quad \left. + \sum_{r\epsilon R} tcw_{pr} x_{pwr}\right) - \sum_{p\epsilon P}\left(\sum_{w\epsilon W} tcd_{pw} b_{pw} + \sum_{r\epsilon R} tcw_{pr}\, q_{pwr}\right)\\
&\quad - \sum_{p\epsilon P}\sum_{r\epsilon R} aba_{pr} y_{pr} - \sum_{p\epsilon P}\sum_{r\epsilon R} abm_{pr}\, t_{pr} - \sum_{p\epsilon P}\left(\sum_{w\epsilon W} abb_{pw}(1 - b_{pw})\right.\\
&\quad \left. + \sum_{r\epsilon R} abw_{pr}\, x_{pwr}\right) - \sum_{p\epsilon P}\left(\sum_{w\epsilon W} abc_{pw}\, b_{pw} + \sum_{r\epsilon R} abw_{pr}\, q_{pwr}\right) + d_i^- - d_i^+\\
&\quad = MP \qquad (17)
\end{aligned}$$

Mp: monthly maximum profit (ideal profit value). Variables d^-_1 and d^+_1 suggest profit lower than MP and more than MP respectively.

*The lead time goal

$$dclt\left(\sum_{p\epsilon P}\sum_{r\epsilon R} y_{pr} + \sum_{p\epsilon P}\sum_{r\epsilon R} slt_p\, t_{pr}\right) + dlt\left(\sum_{p\epsilon P}\sum_{w\epsilon W}\sum_{r\epsilon R} x_{pwr} + \sum_{p\epsilon P}\sum_{w\epsilon W}\sum_{r\epsilon R} q_{pwr}\right) + d_r^- - d_r^+ = MLT \quad (۱۸)$$

The lead time that is the best ideal time is called MLT. The lead time should not be more than MLT. d^-_2 suggests more decrease in time of good dispatch and it is an optimal variable and d^+_2 suggests increase in time of good dispatch so this variable is placed in object function.

*The power of retailer goal

$$\sum_{r\epsilon R} c_r\left(\sum_{p\epsilon P} y_{pr} + \sum_{p\epsilon P} t_{pr}\right) + \sum_{r\epsilon R} c_r\left(\sum_{p\epsilon P}\sum_{w\epsilon W} x_{pwr} + \sum_{p\epsilon P}\sum_{w\epsilon W} q_{pwr}\right) + d_r^- - d_r^+ = MW \quad (۱۹)$$

The most power of retailer that is an ideal one is called MW. Power of retailer should not be lower than MW. Variable d^-_3 suggests decrease in power of retailer and non optimal variable and d^+_3 suggests increase power of retailer and optimal variable so non optimal variable d^-_3 is placed in MIN object function.

*The credit of retailer goal

$$\sum_{r\epsilon R} cp_r\left(\sum_{p\epsilon P} y_{pr} + \sum_{p\epsilon P} t_{pr}\right) + \sum_{r\epsilon R} cp_r\left(\sum_{p\epsilon P}\sum_{w\epsilon W} x_{pwr} + \sum_{p\epsilon P}\sum_{w\epsilon W} q_{pwr}\right) + d_i^- - d_i^+ = MCP \quad (۲۰)$$

Ideal value of customer credit is MCP. Variable d^-_4 shows decrease in retailing credit and a non optimal variable. Variable d^+_4 shows increase in retailing power and an optimal variable. As a result, d^-_4 is placed in ideal object function.

* Product variety goal

$$\mathbf{vw}\sum_{p\epsilon P}\sum_{r\epsilon R} y_{pr} + \sum_{p\epsilon P}\sum_{r\epsilon R} vx_p\, t_{pr} + \mathbf{vb}\left(\sum_{p\epsilon P}\sum_{w\epsilon W}\sum_{r\epsilon R} x_{pwr} + \sum_{p\epsilon P}\sum_{w\epsilon W}\sum_{r\epsilon R} q_{pwr}\right) + d_v^- - d_v^+ = MV \quad (۲۱)$$

The actual goal of the producer achieve to the best combination of customers. Giving suitable services to the customers and product costs are also of producer's objects. so variables d^-_1, d^+_2,d^-_3, d^-_4 and d^-_5 should be minimized in object function. In this equation, W shows weights of each variable. In next section, calculation of these weights will be shown.

$Z=w_1d_1^-+w_2d_2^++w_3d_3^-+w_4d_4^-+w_5d_5^-$ (22)

* Variable weights

Weights of each variable have been calculated using AHP method. five factors of profit, lead time, product variety, power and credit of the retailer have been considered and decision makers are asked to compare

these factors two by two. In following table, two by two comparisons of these factors have been shown. The weights for each criterion may be calculated using the Analytical Hierarchy Process (AHP) by Saaty (1980).

Table 1: two by two comparison of factors

Importance degree of factors	The same preference	Quite preferred	Strongly preferred	Very strong	Extraordinary strong
Preference of profit to lead time					
Preference of profit to power of retailer					
Preference of profit to credit of retailer					
Preference of profit to product variety					
Preference of lead time to power of retailer					
Preference of lead time to credit of retailer					
Preference of lead time to product variety					
Preference of power of retailer to credit of retailer					
Preference of power of retailer to product variety					
Preference of credit of retailer to product variety					

In this study, the above table has been filled separately by three decision makers (selling managers). Data of the table are changed in to a matrix. In this matrix, every factor is compared with the others and its value is placed in the matrix. It is natural that diameter of the matrix is determined by one because preference of each factor to itself is the same. Regarding Saaty range and the table filled by decision maker, the numbers of the matrix are filled as follows:

Table 2: table filled by decision maker

factors	profit	Lead time	Power of retailer	Credit of retailer	Product variety
profit	**1**	**3**	**5**	**7**	**7**
Lead time		**1**	**3**	**3**	**5**
Power of retailer			**1**	**3**	**5**
Credit of retailer				**1**	**7**
Product variety					**1**

In order to fill other cells of the matrix, reversal of above numbers will be placed in the similar position. For example, profit preference to time of good dispatch is 3 so preference of time of good dispatch to the profit is 1/3. The above table is filled as follows:

Table 3: decision maker matrix 1

factors	profit	Lead time	Power of retailer	Credit of retailer	Product variety
profit	**1**	**3**	**5**	**7**	**7**
Lead time	**.33**	**1**	**3**	**3**	**5**
Power of retailer	**.2**	**.33**	**1**	**3**	**5**
Credit of retailer	**.14**	**.33**	**.33**	**1**	**7**
Product variety	**.14**	**.2**	**.2**	**.14**	**1**

Matrix of other decision makers is filled in the same way.

In this stage, opinions of these three decision makers are changed into one opinion then weights of factors are obtained based on them. Geometric mean is used to combine tables of decision makers. The result of calculations is shown in following table 4.

Table 4: matrix of factors' comparisons relative to each other based on opinions of the three decision makers

factors	profit	Lead time	Power of retailer	Credit of retailer	Product variety
profit	**1**	**5**	**5**	**7**	**6**
Lead time	**.21**	**1**	**4**	**4**	**5**
Power of retailer	**.2**	**.24**	**1**	**5**	**6**
Credit of retailer	**.14**	**.24**	**.21**	**1**	**6**
Product variety	**.18**	**.21**	**.18**	**.18**	**1**

Table of decision maker comparison is used to derive priorities. Normalization and weighted mean are used to determine priorities. Normalized values are shown in table 5.

Table 5: Normalized values

factors	profit	Lead time	Power of retailer	Credit of retailer	Product variety	weights
profit	**.58**	**.74**	**.47**	**.41**	**.25**	**.49**
Lead time	**.12**	**.16**	**.4**	**.25**	**.21**	**.23**
Power of retailer	**.12**	**.04**	**.09**	**.28**	**.25**	**.15**
Credit of retailer	**.08**	**.04**	**.02**	**.06**	**.25**	**.09**
Product variety	**.1**	**.03**	**.02**	**.01**	**.04**	**.04**

Column of weights is obtained from above rows' averages.

Results of table 5 show that the profit is in the highest priority then lead time has been obtained the highest weight by decision makers. The high weight of this factor is due to increase of selling to the retailer. Power of the retailer is in higher priority than credit of the retailer because decision makers want to sell to these customers in cash. Credit of the retailer is in next priority and product variety is placed in the last priority.

4. Case study

In this study, Barez Company that is producing tire has been considered and distribution of driving radial tire has been paid attention to approximately, half of radial tires sold to agents relates to Tehran market. There is a good potential in Tehran market and management of distribution causes penetration in market, attraction of more market share and finally increase in producer's profit. At the moment, distribution in this market is done traditionally. Here the price is very important and making decision to buy is based on the price. The wholesalers and retailers supply their products from different producers and are not dependent on producers. In general, Barez Company distributes a small amount of its products to retailers. The retailers can't buy Barez directly due to increase in transportation cost from Factory to Tehran and lack of enough space for warehousing the products. They also will be under pressure because of lack of enough space and lower financial ability than wholesalers so they can not sell their products as same price as the wholesalers and can not gain the expected profit. In this study, three producers of Barez, Dena and Kavir have been used. 4 wholesalers were chosen that distribute tires of these producers. 40 retailers were also chosen among customers of Barez and customers of wholesalers.

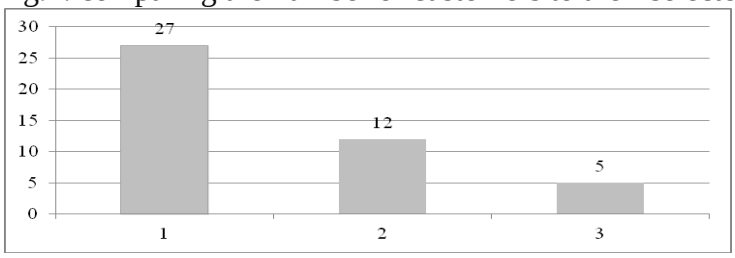
The obtained data are as follows:

- Monthly demand of 44 retailers and wholesalers during 12 months
- Transportation costs of customers from producers and distribution center (including number of vehicles and the cost of each vehicle) during 12 months
- warehousing costs (rent of warehouse, warehousing costs and salary of employers) during twelve months
- Lead time from the supplier (producers, distribution center and wholesalers) during 12 months
- Power and credit of the retailer evaluated by suppliers (producers, wholesalers) during 12 months
- Evaluation of product variety by customers during 12 months
- selling margin of customers (retailers and wholesalers) during 12 months

5. Results

The above model is implemented twelve times per year (once a month). Regarding demand changes, the selling managers will control and observe distribution options chosen by the customers. The managers want to supply customer who at least demand 6 times per year. In this study, the options chosen by 44 customers have been studied during one year. Fig.1 shows a graph with the number of customers in Y axis and number of options chosen by customers in X axis. For example, of 44 customers, 27 ones have chosen one supply option. 12 customers have used two supply options and 5 customers have used three supply options. Most of customers (39 customers) have chosen one or two supply options.

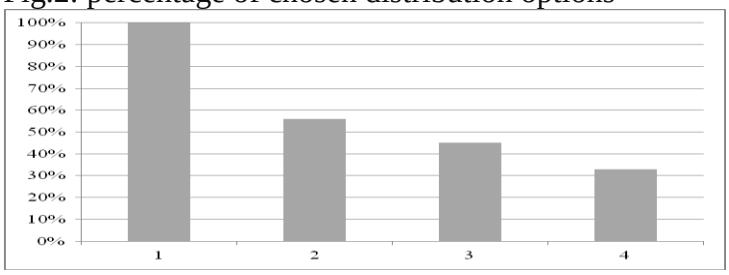
fig.1: comparing the number of customers to their selected solutions



Although in this section only three distribution options were studied, in fact, number of these options is more than three. For example, a wholesaler can supply his needs during the year by one of producers, two producers, three producers or combination of producers and distribution center. A retailer also can supply his demands by producers, wholesalers and distribution centers or a combination of these three. For example, if the customer has chosen two supply options, it may be possible that he has used one option for two times and

another option for ten times. In following figure, percentage of chosen options has been shown. If one option is chosen by customers during twelve years, this option will be chosen by 100%. When two options are chosen by the customer, the selective option will be chosen by 56%.

Fig.2: percentage of chosen distribution options



Regarding 44 customers considered in this study, 7 retailers supply their demand from Barez distribution center. Barez Company supplies 4 wholesalers and one retailer and 4 wholesalers supply 33 retailers. In this study after implementing this model by GAMS software (12 times per year) and regarding 5 factors of profit, power of retailer, credit of retailer, lead time and product variety in object function of goal programming, optimum distribution to customers will be determined. The GAMS model included 10 continuous variables and 1633 discrete variables and 681 equations. The model run with the Dell Vostro 1520, 2.6 GHz CPU with 4 GB RAM, of which about .4 s are spent processing the data.

Optimal distribution to customers is as follows: 34 retailers are supplied by warehouse of Tehran Barez. 8 retailers are supplied by Dena distribution center and 17 retailers by Kavir distribution center. Regarding high purchases, two retailers are supplied directly by the factory. Due to high demand of retailers from Dena and Kavir, direct dispatch from these producers to retailers is not optimal. 5 retailers supply their needs by wholesalers from Barez Co. 4 retailers also supply their needs by wholesalers from Dena and Kavir. 4 wholesalers are supplied by Barez and one wholesaler by Dena and Kavir. Regarding optimal distribution, retailers with low power and credit are left for wholesalers and the company does not tend to supply these customers. If chosen retailers have high capability, they will be supplied by distribution centers and it causes company profit and market share to increase gradually. Mean time of good dispatch from Barez, Kavir and Dena is 15, 18 and 17 days respectively, mean lead time from distribution center is two days and mean lead time from wholesalers is 3 days.

Product variety from Barez, Kavir and Dena is 13, 10 and 10 respectively. Product variety from distribution center is 8 and 5 from wholesaler.

In table 7, ideal results have been compared to obtainable results.

Table 6: comparing ideal and obtainable objects

objects	Ideal object	Obtainable object	difference
Profit	370197496	365300000	1%
Credit of retailer	675	661	2%
Variety product	970	712	27%
Power of retailer	648	636	2%
Lead time	120	128	7%

As shown in above table, obtainable profit is close to ideal profit and this goal has been approximately realized. So the retailer will gain a good profit by supplying from Tehran distribution center. Obtainable object and the credit and power of retailer are close to the ideal object. So the company can keep capable

customers and leaves low capable customers for wholesalers. In an ideal condition for increasing product variety, the customers should supply their needs from suppliers. Regarding other objects, obtainable object is 27% lower than ideal object. On the other hand, increase of variety in regional warehouse increases warehouse costs so that it is not cost effective for the supplier. Lead time is 7% lower than ideal object. This percent difference is due to lack of tendency of the company for meeting the needs of customers with low power. The customers with low power are supplied by wholesalers.

5.1 Weight sensitivity analysis

In this study, sensitivity analysis has been done for two factors of profit and decrease in time of good dispatch. Value of object function by introduction of parameter θ is as follows (Mehregan,2007) :

$$Z=(W_1-\Theta)d_1^-+(W_2+\Theta)d_2^+ +W_3d_3^-+W_4d_4^-+W_5d_5^- \quad (23)$$

W values are placed in goal programming regarding results obtained from AHP method.

$$Z=(0.49-\Theta)d_1^-+(0.23+\Theta)d_2^+ +0.15d_3^-+0.09d_4^-+0.04d_5^- \quad (24)$$

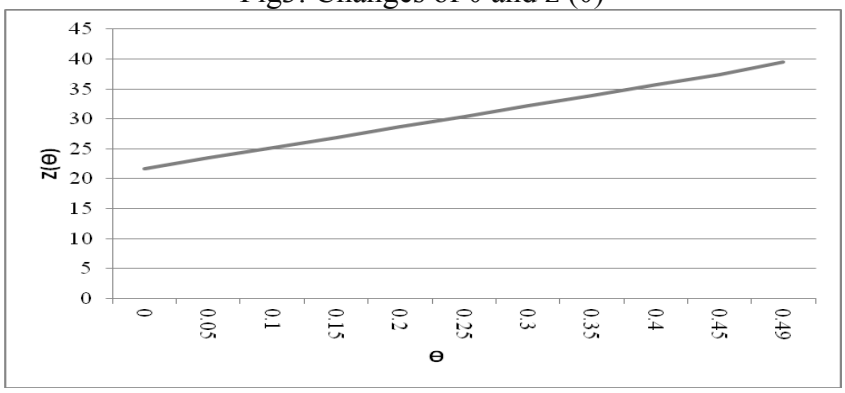
At first, θ value is zero then gradually its value is increased. So a table for θ values and value of object function is obtained by increasing θ and obtaining value of ideal object function. θ values and z (θ) value are shown in table 8.

Table 7: θ values and z (θ) value

Θ	Z(Θ)
0	21.69
.05	23.44
.1	25.19
.15	26.94
.2	28.69
.25	30.44
.3	32.19
.35	33.94
.4	35.69
.45	37.44
.49	39.45

Changes of θ and z (θ) are shown in fig 3.

Fig3: Changes of θ and z (θ)



This diagram shows that the selective weights are suitable for two factors of profit and lead time. Function value is minimal per values determined in goal programming function.

5.2 Optimal distribution value in distribution network

Regarding the network suggested in this model, the ratio of supplying from Tehran warehouse to supplying from Barez Company for Tehrani customers is 24% to76% during twelve months. At the moment, this ratio is 6% to 94%. These results are shown in figs 4 and 5.

Fig.4: current ratio in present network

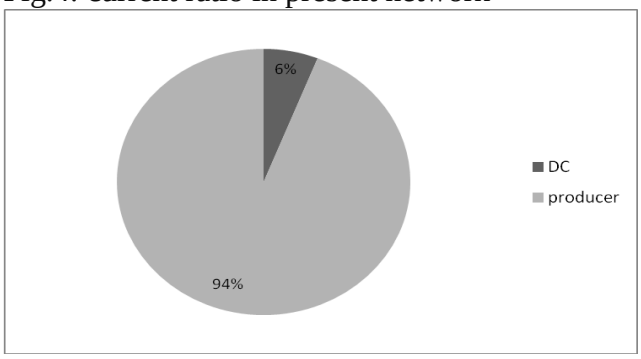
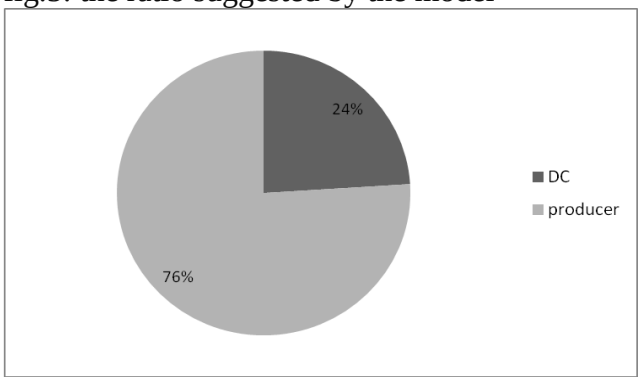


fig.5: the ratio suggested by the model



In this study, distribution of driving radial tires has been considered that have a large demand in Tehran market. With 24% distribution of driving radial tires from Tehran distribution center, these products can be sold by a 7% price increase. Transportation cost to Tehran warehouse and cost of Tehran warehouse are 1.2% to selling price. The profit obtained from monthly selling is 3300 million rials. Selling from Tehran distribution center to retailers makes wholesalers dissatisfied and purchase of wholesalers may reduce. 24% distribution of products to retailers is a difficult task and it will gradually increase.

In order to reduce costs, the customers want to supply their needs from one source. Suitable supply of retailers by Tehran distribution center, increase of product variety, decrease lead time and make increase of market share of Barez tires. By approaching retailers, we can understand information of competitions regarding the price, new products and their selling conditions then we can make better decisions by quick transition of information. Training selling force can be better done and they will be familiar with consumers' needs and up-to-date products and services are presented.

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