Estimation of Signal Parameters in Maritime Environment Using Extended Sub-Space Based Approach

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Abstract
During the past few years there has been an increasing interest in applying sub-space methods to a wide range of Signal Processing and RADAR system theory problems, due to their computational simplicity. However, these techniques are not efficient to resolve closely parameters of signals embedded in maritime environment characterised by correlated non-Gaussian clutter, which is modelled in this paper as a compound Gaussian process. We propose in this paper an extended sub-space based approach for frequency estimation of a signal in K-distributed clutter, using the Fixed Point Covariance Estimation of the clutter. Numerical results obtained by Monte Carlo Simulation demonstrate the effectiveness and efficiency of the proposed method.

Keywords: RADAR, High resolution algorithms, Sub-Space Methods, Maritime environment, K-distribution.

1. Introduction
In the field of RADAR, SONAR and mobile communication, there has been a growing interest in developing high-resolution signal parameters estimation techniques, such a problem arises in many practical fields and has already received considerable attention in the signal processing literature.

For additive white Gaussian noise, signal parameters can be estimated using the sub-space methods, among them MUSIC (MUltiple SIgnal Classification) (R. Schmidt, 1986), ESPRIT (Estimation of Signal Parameters via Rotational Invariance Techniques (ROY, 1989), and MATRIX PENCIL (Y. Hua et al, 1990). Higher order statistics based methods can be used in the case of Gaussian additive noise and also when the signal is contaminated with coloured Gaussian noise, the key lies in that all the Gaussian noise cumulants of order greater than two are equal to zero. Therefore, the noise can be suppressed in the cumulant domain (C. K. Papadopoulos et al, 1990). Spatial smoothing based techniques introduced firstly by Evan et al and extensively studied by Shan et al (1985) and Pillai et al (1989) are relatively more effective when signals are coherent or highly correlated.

The methods given above all assume that the additive noise is Gaussian process or the noise covariance matrix is known in advance. On the contrary, in many practical situations like the maritime environment, the additive noise is Non–Gaussian and a priori estimate of the noise covariance matrix is not available. Therefore these high resolution sub-space methods will suffer severe performance degradation. A program of work looking at the Non-Gaussian nature of sea clutter has resulted in a model known as the compound K-distribution, which is a particular case of a SIRV (Spherically Invariant Random Vector), this model, has been presented and applied in previous publications.

To overcome the limitation of the above sub-space algorithms against Non-Gaussian clutter, we extend them in this paper, to the case of K-distributed clutter. Our approach can be decomposed into three steps: The first step is to estimate the parameters of the K-distribution which descript the marine environment and the sea state, the second step is to estimate the clutter covariance matrix using the Fixed Point Estimator, and the last step is to apply the sub-space methods to the whitened matrix reconstructed from the eigen-value decomposition of the covariance matrix of the clutter.

The rest of the paper is organized as follows. A short description of the maritime environment is introduced in section 2. In section 3, we expose the formulation of the problem, this section includes a description of RADAR signals, a brief review of the SIRV and K-distribution process and finally a study of the K-distribution parameters. In section 4, we detail our proposed algorithm. Results of our simulation, based on Monte Carlo technique, are presented and interpreted in section 5.

2. Maritime environment characteristics

In marine environment signals are non stationary and are embedded into a Non-Gaussian noise, which can be modelled by various distributions such as Log-normal, Weibull and K-distribution, which is known by its capability to modelling Non-Gaussian clutter and also by its adaptability to modelling coherent signals. In fact:

The important factor that perturbs the RADAR waves intercepted by the receiver antenna is the clutter, principally sea clutter, the nature of surface roughness determines the proprieties of RADAR echo. The roughness of the sea surface is normally characterized in terms of two fundamental types of waves:

- The gravity waves, which describe the macrostructure of the sea surface, can be subdivided into sea and swell.
- The capillary waves, which are usually caused by turbulent gusts of wind near the surface , they describe the microstructure of the sea surface dynamics, and are affected by a variety of forces giving a Non-Stationary Spatio-Temporel structure on the RADAR observable, they are summarized here:
    - Gravitational and rotational forces, which permeate the entire fluid, with large scales compared with most other forces.
    - Thermodynamic forces, such as radiative transfer, heating, cooling precipitation and evaporation.
    - Mechanical forces, such as surface wind stress, atmospheric pressure variations and other mechanical perturbations.
    - Internal forces, pressure and viscosity, exerted by one portion of the fluid on other parts.

These forces give a significant structure to the sea surface. When the wind blows, it generate small ripples, which grow and transfer their energy to large waves, at some point the waves become large enough to break and this redistributes the wave energy further. When the wind has been blowing for some time, equilibrium is established between the input of energy and its dissipation, there is then a wide spectrum of waves propagating on the sea, which can be added to by swell, travelling into the area from remote rough weather. All of these waves and the breaking events are reflected in spatio-temporel variation of clutter returns, this gives a Non-Gaussian amplitude statistics of sea clutter, which vary considerably, depending on the prevailing sea and weather conditions, the viewing geometry and the RADAR parameters (Keith D Ward et al, 2006).

At low grazing angle (less than 10°), the scattering mechanisms are much more complex, caused by factors such as, shadowing, diffraction and interference.

For RADARs that have a high spatial resolution, the returns are often described as becoming spiky, in this case, the spiky clutter has a Non-Gaussian nature and has been modelled, according to several researchers, as having K-distribution PDF (Probability Density Function).

3. Problem formulation

3.1. RADAR signals

Signal received by the RADAR is composed of the target signal and the clutter, the matrix representation of this composed signal after sampling ($t = n.T_s$ with T_s is the sampling period) can be written:

$$Z[n] = Y[n] + C[n] \qquad (1)$$

$Y[n] = \alpha_0 e^{j2\pi f_D n}$ is the Target signal, where f_D is the Target Doppler Frequency and $\alpha_0 = A_0 e^{j\theta_0}$ is the complex amplitude, with A_0 and θ_0 are the amplitude and initial phase of the target signal, they are modelled as unknown parameters to account for both propagation effects and target scintillation. In this work we consider that the target amplitude fluctuates according to the Swerling 1 model (Eyung W. Kang, 2008).
$C[n]$ is correlated Non-Gaussian clutter modelled as a compound-Gaussian Process.

Detection, localization and identification of the targets are provided by eliminating or reducing the effect of clutter, and thereafter determine targets parameters.

The clutter can be static with determined direction (mountains, terrestrial surface and all fixed objects); it can also be at origin of dynamic targets (vague, rain, snow or any non-desired dynamic object). The static clutter is easily filterable by using MTI technique (Moving Target Indicator), unlike the dynamic clutter coming mainly from the marine surface, we have two cases:

- Target's radial velocity is sufficiently high compared to clutter radial velocity, the coherent RADAR employing Doppler processing can distinguish targets from clutter by using simply the MTI technique.
- Targets of interest have Doppler shifts that are not significantly different from the Doppler spectrum of the clutter, this case requires in addition to MTI techniques, detailed understanding of the characteristics of the Doppler spectrum, for all environment conditions and need others techniques of filtering, such as signal whitening , adaptive filters and sub-space methods.

3.2. Model of the K-distributed clutter

Many authors have investigated the fit of sea clutter data to the compound formulation described by the SIRV process

$$C = \sqrt{\tau} x \qquad (2)$$

so that $x = \zeta N(0, R_c)$, x is commonly named speckle component , which covariance matrix R_c is here estimated by Fixed Point Estimator. τ so called texture, is a real positive random process, this representation is widely used to model the RADAR clutter. When the texture is Gamma distributed, the multivariate distribution of the clutter vector is reduced to the K-distribution. For high resolution and angles of observation lower than 10°, clutter sea amplitude is characterized by a Non-Gaussian nature and by the presence of spikes, it is well modelled by the K-distribution, as it is described here was developed by Ward, it interprets the sea clutter like containing two components:
A fast component x described by the distribution of Rayleigh, which models the speckle and a slow component τ characterized by a Gamma distribution which models the texture.
Sea clutter is thus represented by the speckle modulated by a varying underlying mean level or local power. Its PDF is given by:

$$P(E) = \int_0^\infty dx P(E \mid x) P_c(x) \qquad (3)$$

With :

$$P(E \mid x) = \frac{2E}{x} \exp\left(\frac{-E^2}{x}\right), 0 \le E \le \infty \qquad (4)$$

Is the PDF of the speckle and

$$P_c(x) = \frac{\mu^{\nu}}{\Gamma(\nu)} x^{\nu-1} \exp(-\mu x), 0 \le x \le \infty \quad (5)$$

is the PDF of the local power (texture) characterized by two parameters: the Shape : ν and the Scale : μ which depend on the climatic conditions and RADAR parameters.
This gives a K-distributed model to the sea clutter:

$$P(E) = \frac{4\mu^{(\nu+1)/2} E^{\nu}}{\Gamma(\nu)} K_{\nu-1}(2E\sqrt{\mu}) \quad (6)$$

For the intensity $z = E^2$:

$$P(z) = \frac{2\mu^{(\nu+1)/2} z^{(\nu-1)/2}}{\Gamma(\nu)} K_{\nu-1}(2\sqrt{\mu z}) \quad (7)$$

The following figure represents the K-distribution for different values of ν [22].

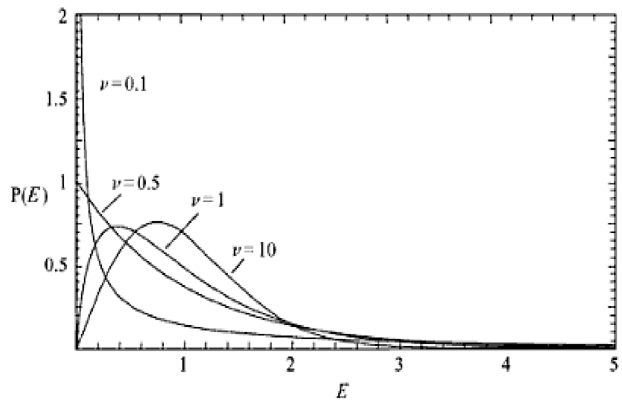

Fig.1. The K-distribution for various values of ν .

3.3. Estimate of the parameters of the K-distribution

In practice, the parameters of the distributions of the amplitudes, the properties of correlation and the statistical proprieties of the environment are not known; the RADAR must thus estimate them, in order to well model the clutter and to adapt its treatments and also to regulate its threshold of detection, on a suitable level for the conditions met in real time.
In this regard, robust models were developed, particularly to estimate the parameters of the K distributed clutter.

3.3.1. Parameter Shape:

This parameter gives statistics information of the amplitude of received signal (the less is the value of this parameter, the spikier is the sea clutter). An empirical model is developed by Ward, connecting this parameter to various characteristics of the RADAR, viewing geometry and of the environment; it is summarized by the following equation [22]:

$$\log_{10}(\nu) = \frac{2}{3}\log_{10}(\varphi) + \frac{5}{8}\log_{10}(A_c) + \delta - k_{pol} \quad (8)$$

with:
- ν : shape
- A_c : RADAR resolved area
- φ : grazing angle in degree (0.1°< φ <10°)

- δ used for swell directions dependence:
  - δ =-1/3 for up or down swell directions
  - δ = +1/3 for cross-swell directions
  - δ = 0 for intermediate directions or when no swell exists.

The value of δ can be determined by $\delta = -\frac{1}{3}\cos(2\theta_{SW})$, with θ_{SW} indicate the direction of the swell.

- k_{pol} describe the effect of polarization, k_{pol} = 1.39 for a vertical polarization and k_{pol} =2.09 for a horizontal polarization.

3.3.2. Parameter: Scale

The Scale parameter μ gives information about the power of the received signal (the less is the value of the scale parameter, the more powerful are reflected signals from the sea surface), it is calculated from the reflectivity of the clutter as well as the parameters of the RADAR:

$$\mu = \sqrt{\frac{4\nu}{P_t G_t^2 \frac{\lambda^2 f^4}{(4\pi)^3 R^3}(\sigma_0 \theta_B \frac{c\tau_{pulse}}{2})}} \qquad (9)$$

with:
- P_t : Transmitted power
- G_t : Transmit gain
- λ : RADAR wavelength
- f^4 : Two-way antenna pattern value
- R : Slant range to the clutter cell
- θ_B : Antenna pattern beam-width
- τ_{pulse} : Pulse width
- c: Speed of light
- σ_0 : Mean clutter reflectivity value.

4. Extended Sub-space based methods using Fixed Point Covariance Estimator

4.1.Fixed Point Covariance Estimator

Gini et al. (2000) derived an approximate Maximum Likelihood estimate of the covariance of a SIRV process, as the solution of the following equation:

$\hat{M} = f(\hat{M})$ Where f is given by:

$$f(\hat{M}) = \frac{m}{N}\sum_{i=1}^{N}\frac{c_i c_i^H}{c_i^H \hat{M}^{-1} c_i} \qquad (10)$$

$\hat{M}$ is normalized according to $Tr(M^{-1}\hat{M}) = m$ and N is the number of secondary data.

It has demonstrated that $f(\hat{M})$ does not depend on the texture of the SIRV process, but only on the Gaussian vectors x_i 's (F. Gini et al, 2002). Consequently f can be written as:

$$f(\hat{M}) = \frac{m}{N}\sum_{i=1}^{N}\frac{x_i x_i^H}{x_i^H \hat{M}^{-1} x_i} \qquad (11)$$

$\hat{M}$ is constructed numerically by solving this iterative algorithm:

$$\hat{M}(k+1) = \frac{m}{N}\sum_{i=1}^{N}\frac{x_i x_i^H}{x_i^H (\hat{M}^{(k)})^{-1} x_i}\;; k \in \mathrm{N} \quad (12)$$

This algorithm converges to $\hat{M}$ whatever the initial matrix $\hat{M}(0)$. In consequence, the identity Matrix is chosen for initialising $\hat{M}$.
For a Toeplitz matrix (known by its effectiveness to model a SIRV process) fully defined by the correlation parameter ρ and defined by the following structure:

$M_{ij} = \rho^{|i-j|}$ For $i \le i, j \le m$ and $0 \le \rho \le 1$

For $m=10$, $N=20$, starting point $M_0 = I_m$ and a high correlation $\rho = 0.9$, which are the parameters used in our simulation (Fig. 2), the convergence criterion Crv is reached around 70 iterations

$$Crv(k) = \frac{\left\| \hat{M}_{k+1} - \hat{M}_k \right\|}{\left\| \hat{M}_k \right\|} \quad (13)$$

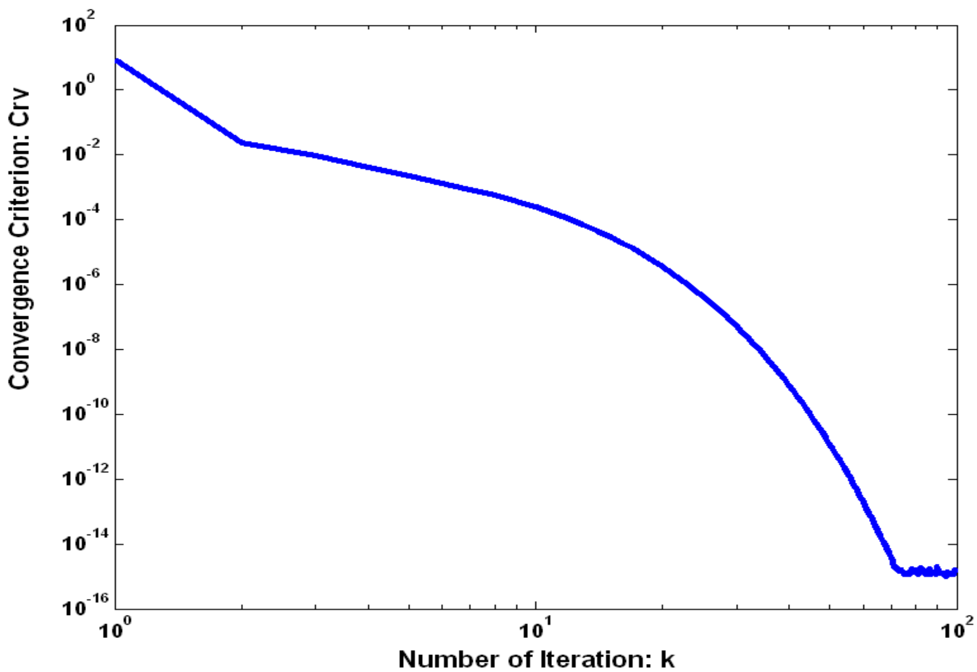

Fig.2. Convergence to the Fixed Point for $\rho = 0.9$

The choice of the Fixed Point estimator of the clutter covariance matrix is motivated by its good statistical performance (Consistency, Unbiasedness and Asymptotic Gaussianity) (Frédéric Pascal et al, 2006) and by the fact that this estimator is applicable to practical cases, since it is based on vectors of K-distributed clutter, estimated from the environmental and climate conditions as well as RADAR properties.
4.2.Extended Sub-Space Methods

The process of our extended sub-space algorithms is to form the sample covariance of the clutter, by using the Fixed Point Estimator described in the previous section, whitening the clutter by exploiting the Eigen-decomposition of the estimated second order information of the clutter (Rc). The Non-Gaussian clutter problem is transformed to the famous problem of estimating signal parameters in white Gaussian noise, and thus we can use the standard sub-space methods to the whitened data and conclude original signal parameters.

The eigen-decomposition of the clutter covariance Matrix estimated by Fixed Point algorithm gives:

$$Rc = E_c \Lambda E_c^H \qquad (14)$$

Consider the whitening Matrix:

$$W = \Lambda^{-\frac{1}{2}} E_c^H C \qquad (15)$$

For a zero mean value of the clutter the statistic values of this transformation are:

- Expectation: $$E(W) = \Lambda^{-\frac{1}{2}} E_c^H E(C) = 0 \qquad (16)$$
- Covariance: $$E(WW^H) = E\left(\Lambda^{-\frac{1}{2}} E_c^H C \left(\Lambda^{-\frac{1}{2}} E_c^H C \right)^H \right)$$

$$= E\left(\Lambda^{-\frac{1}{2}} E_c^H C C^H E_c \Lambda^{-\frac{1}{2}} \right)$$

$$= \Lambda^{-\frac{1}{2}} E_c^H E(CC^H) E_c \Lambda^{-\frac{1}{2}}$$

$$= \Lambda^{-\frac{1}{2}} E_c^H Rc E_c \Lambda^{-\frac{1}{2}}$$

$$= \Lambda^{-\frac{1}{2}} E_c^H E_c \Lambda E_c^H E_c \Lambda^{-\frac{1}{2}}$$

$$= \Lambda^{-\frac{1}{2}} \Lambda^{\frac{1}{2}}$$

$$E(WW^H) = I \qquad (17)$$

Multiplying the received signal by $\Lambda^{-\frac{1}{2}} E^H$, we obtain:

$$\Lambda^{-\frac{1}{2}} E^H Z = \Lambda^{-\frac{1}{2}} E^H Y + \Lambda^{-\frac{1}{2}} E^H C \qquad (18)$$

In other words:

$$Z' = Y' + W \qquad (19)$$

Given the Matrix W, with zero mean and identity covariance, we can therefore use the standard sub-space based methods.

5. Simulations and results

To examine the performance of our approach in the case of marine environment with high resolution RADAR and small greasing angle, it was considered a sinusoid plunged in a K-distributed clutter.

The steps of our simulation are as follows:

1- To represent the environmental conditions and RADAR parameters we fix the parameters of the K-distribution (ν, μ);

2- To represent the clutter we generate random numbers according to the K-distribution law;

3- Clutter covariance matrix is estimated by Fixed Point Estimator;

4- The received signal is whitened by exploiting the clutter covariance matrix;
5- To detect the frequency of the sinusoid, ROOT-MUSIC, TLS-ESPRIT and TLS MATRIX PENCIL algorithm are applied to the whitened signal;
6- Monte Carlo simulation based on 1000 repetitions is used to calculate the MSE (Mean Square Error) and deduce performances of these extended sub-space algorithms, libelled FP-ROOT-MUSIC, FP-TLS-ESPRIT and FP-MATRIX-PENCIL, by using a benchmark with the Hybrid Cramer Rao Lower Bound used by Fulvio Gini et al (2000).

In order to confirm the improvement obtained by the proposed approach, we evaluate the Mean Squared Error (MSE) of frequency estimation versus the SCR, which is varying from 0 to 30dB by steps of 5 dB. The result is plotted in Fig.3 and compared with the Hybrid Cramer Rao Lower Bound estimator.

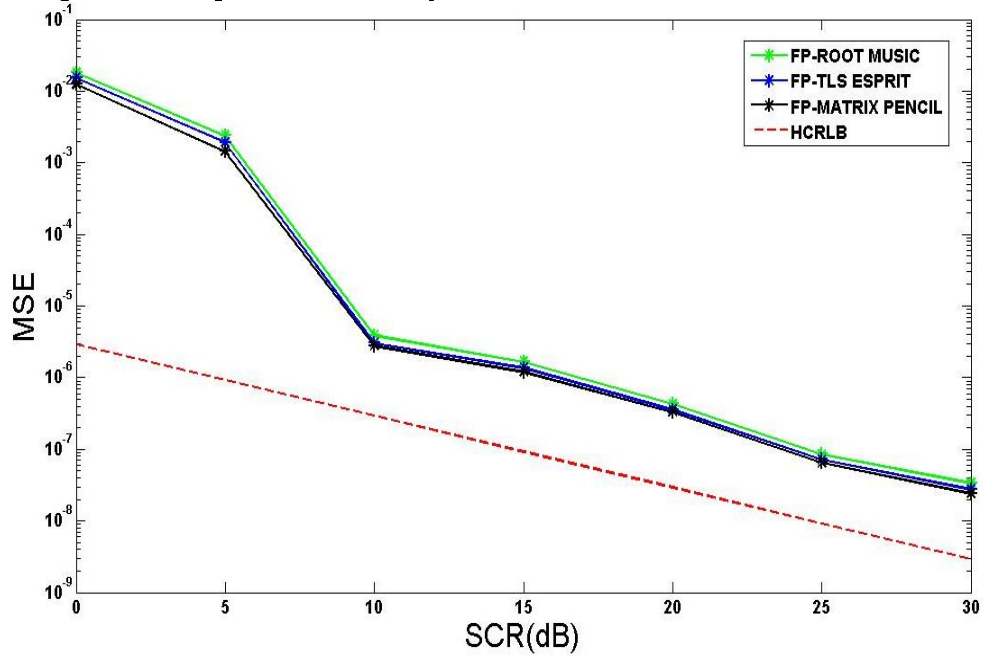

Fig.3. MSE and HCRLB depending on the signal/Clutter ratio for N=16, $\nu=1$, $\rho=0.9$ and $f_D=0.25$.

Fig.3 shows the case of a spiky ($\nu=1$) and correlated clutter ($\rho=0.9$), the MSE decrease rapidly when SCR ≤ 10dB, for SCR>10dB it decrease approximately as 1/SCR like the HCRLB low. We can conclude that for low values of the parameter shape, which represents the case of agitated marine sea surface and spiky clutter, our algorithm is appropriate to resolve the frequency parameter of a sinusoid in K-distributed clutter, it converges more considerably to the Hybrid Cramer Rao Lower Bound as soon as SCR increase.
Fig.4 represents M.S.E depending on the number of samples: N

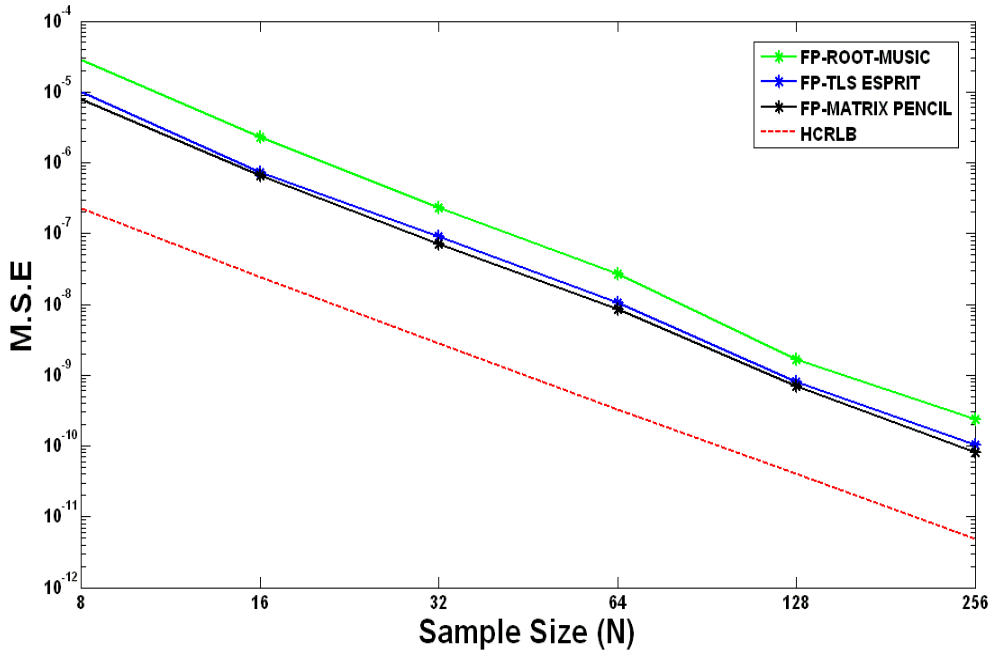

Fig.4. MSE and HCRLB depending on the number of samples for SCR=10dB, $\nu=1$, $\rho=0.9$ and $f_D=0.25$.

We observe in Fig. 4 that M.S.E decrease as soon as the number of samples increases.

According to our simulations, we can conclude that when the additive clutter is Non-Gaussian with unknown autocorrelation matrix the FP-Subspace Methods offer a significant improvement in the estimation performance. Furthermore, our approach seems to be more robust to additive K-distributed clutter than the standard Subspace Methods.

6. Conclusion

In this article we proposed an extended sub-space based methods algorithm, founded on the Fixed Point Estimator of the clutter covariance, to estimate parameters of RADAR signals plunged in the marine environment. The complexity of the Non-Gaussian clutter is reduced by applying a clutter covariance whitening mechanism. Simulation results demonstrated the effectiveness of our algorithm to detect the frequency of a sinusoid against the maritime environment modelled as a compound process. Furthermore, according to the results of our simulations seen in section 5, we concluded its ability to resolve signals frequencies for different climatic conditions, even for the case of surface sea very agitated and strong clutter amplitude.

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