

## Uniform Relative Reciprocal Velocity in Lorentz -Einstein Transformations

*Faisal.A.Al-Bassam*

Babylon University, College of Science for Women, Laser Physics Department, Irqa

*E-mail:albassam1799@yahoo.co.uk*

### Abstract

The relative dimension velocity were converted to the relative time velocity in all equations by using the reciprocal of the velocity that equal to the negative of time velocity. The equations of the relative time velocity of two moving particles are in the same and opposite direction. The resultant of time velocity of two moving particles travel with right and less right angle ,while the resultant of time velocity in three-components. The relative uniform translational and rotational time velocities are derived from the known dimension velocity equations. The Lorentz- Einstein transformations are converted by the principle formula(reciprocal ). In addition to, the classical relative of elapsed time was produced in a different directions of two moving particles, so that the time values was postulate to verify the time of moving one particle or more. The new relative reciprocal equations give an obvious and complementary idea about the classical relative motion, whenever the special relativity of moving particles with very high time velocities are also verified.

Key wards: reciprocal, relative, velocity, Lorentz- Einstein transformation, translation, rotation.

## Introduction

The universe space and time are reciprocally throughout related. The name “Reciprocal” has been applied to the system of theory based on the “motion” concept of the nature of the universe. The reciprocal postulate provides a good example of the manner in which a change in the basic concept of the nature of the universe alters the way in which we apprehend specific physical phenomena. Motion is defined as the relation of space to time.

Inasmuch as the postulates deal with space and time in precisely the same manner, aside from the reciprocal relation between the two. We know time only as a progression, a continual moving forward, whereas space appears to us as an entity that “stays put.”

Quite the contrary, time is scalar in this space velocity equation (and in all of the other familiar vectorial equations of modern physics; equations that are vectorial because they involve direction in space) *irrespective of its dimensions*, because no matter how many dimensions it may have, time has no direction in *space*.

Regardless of its dimensions, time cannot be a vector quantity in any equation such as those of present-day physics, in which the property, which qualifies a quantity as vectorial, is that of having a direction in space.

Inasmuch as everything physical in a universe of motion is a motion—that is, a relation between space and time, measured as speed—and, as we have just seen, the properties of space and those of time are identical, aside from the reciprocal relationship. It follows that every physical entity or phenomenon has a reciprocal any increase in vectorial speed of the s/t pseudo scalar, beyond the unit level crossing the c-speed, boundary so-to-speak is tantamount to a decrease in the vectorial speed of a t/s pseudo scalar. Moreover, we can see that, what would appear to be an increase of s/t vectorial speed, is actually a decrease in t/s vectorial speed, which completely transforms the s/t pseudo scalar into the t/s pseudo scalar and vice-versa.

Kinematics treats the geometry of motion without taking into account their inertia[1]. A relative concept must always be referred to a particular frame of reference, chosen by observer. Different observers may use different frame of reference. It is important to know how observations made by different observers are related[2]. In solving problems of mechanics, it is often more expedient to consider the motion of a particle simultaneously with respect to frames of reference, one of which is assumed to be fixed and the other moving in some specified way with reference to the first[3]. The motion performed in this case by the

particle is called resultant, or combined motion. The method of resolving a motion into simpler motions by introducing a supplementary moving frame of reference is widely employed in kinematics calculations, thereby underling the practical value of the theory of resultant motion considered in this[4]. The stationary and motion are relative concepts that serve as frame of reference [1]. An event of motion has not only position; it also has a time of occurrence [5]. In pure geometry the theories of similar (reciprocal theorem of Betti and Raleigh) reciprocal and inverse figures (reciprocal diagram) have led to many extensions of science (e.g. the refractive index being proportional to the velocity or the reciprocal of velocity) [6]. The concepts generalize to time - varying and to vector – valued Morse functions [7]. It sometimes uses the reciprocal lattice for crystal structure. (Lima Siow) formulated equivalent principle that the kinetic acceleration is equal to the potential acceleration  $(d^2r/dt^2 = -d\Phi/dr)$  [8].

This theoretical study referred to relative reciprocal of dimension velocity (time velocity) with respect to frame of reference and observer of moving body, due to Galilean transformation, also related to Lorentz(Einstein) transformation. These equations give a new general formula of relative motion of the particle with respect to fixed point or relative moving point in addition to the known relative equations. It is regarded an integrated equations to know the relative motions of bodies. The relative reciprocal velocity of moving particles with respect to observer:

**a)The relative time velocity of two particles:**

The relative dimension velocity of a particle with respect to another in the same direction is equal to the difference of two dimension velocities. So the relative dimension velocity  $v_{x(ab)}$  of a particle a have velocity  $v_{x(a)}$  with respect to a particle b have velocity  $v_{x(b)}$  is given by [1]:

$$v_{x(ab)} = v_{x(a)} - v_{x(b)} \tag{1}$$

It was known that  $v_x = -\frac{1}{v_t}$ , (the displacement velocity is equal to negative of reciprocal time velocity)

[5], substitute in eq.(1) :

$$-\frac{1}{v_{t(ab)}} = -\frac{1}{v_{ta}} + \frac{1}{v_{tb}} \tag{2}$$

Or

$$v_{t(ab)} = -\frac{v_{ta} v_{tb}}{v_{ta} - v_{tb}} \tag{3}$$

This equation represents the relative time velocity of two particles travel in the same direction,  
for ex. if:

$v_{ta} = -0.2s / m, v_{tb} = -0.5s / m, then, v_{t(ab)} = -0.333s / m, v_{x(ab)} = 3m / s$  If two particles travel in opposite direction, the equation becomes:

$$V_{x(ab)} = V_{x(a)} + V_{x(b)} \quad (4)$$

And the relative time velocity of the two particles is:

$$v_{t(ab)} = \frac{v_{ta} v_{tb}}{v_{ta} + v_{tb}} \quad (5)$$

For ex. ,if:

$$v_{ta} = -0.2s / m, v_{tb} = -0.5s / m, then, v_{t(ab)} = -0.1428 s / m, and, v_{x(ab)} = 7m / s ,$$

**b) Relative time of two traveling particles:**

The relative time of two traveling particles in the same direction obtained from equation (3) and  $v_t=dt/dx$  [3]:

$$v_{t(ab)} = dt_{(ab)}/dx_{(ab)} \quad (6)$$

$$\frac{dt_{ab}}{dx_{ab}} = - \frac{\frac{\partial t_a \partial t_b}{\partial x_a \partial x_b}}{\frac{\partial t_a}{\partial x_a} - \frac{\partial t_b}{\partial x_b}} \quad (7)$$

By integrating the above equation, the results:

$$dt_{ab} = \frac{\partial t_a \partial t_b dx_{ab}}{\partial x_a \partial t_b - \partial x_b \partial t_a} \quad (8)$$

By dividing the right hand side with  $\partial t_a \partial t_b$  obtained:

$$dt_{ab} = \frac{dx_{ab}}{v_{xa} - v_{xb}} \quad (9)$$

If the distance  $dx(ab) = \partial x_a = \partial x_b$ , and conciliate from eq.(8) the results:

$$t = \frac{\partial t_a \partial t_b}{\partial t_b - \partial t_a} \quad (10)$$

For ex.,if:  $\partial t_a = 2s, \partial t_b = 5s, t = 3.3333s$ , that is the time difference between two particles travel in the same direction at equal distance.

In opposite direction, eq. (7) becomes:

$$\frac{dt_{ab}}{dx_{ab}} = \frac{\frac{\partial t_a \partial t_b}{\partial x_a \partial x_b}}{\frac{\partial t_a}{\partial x_a} + \frac{\partial t_b}{\partial x_b}} \quad (11)$$

With the same method :

$$dt = \frac{\partial t_a \partial t_b}{\partial t_a + \partial t_b} \quad (12)$$

**c)The resultant of time velocity of a particle travel with right angle[2] :**

Since  $v_{x(ab)}^2 = v_{xa}^2 + v_{xb}^2$

$$\left(-\frac{1}{v_{t(ab)}}\right)^2 = \left(-\frac{1}{v_{ta}}\right)^2 + \left(-\frac{1}{v_{tb}}\right)^2 \quad (13)$$

So

$$v_{t(ab)} = \frac{v_{ta} v_{tb}}{\sqrt{v_{ta}^2 + v_{tb}^2}} \quad (14)$$

$$v_{t(ab)} = \frac{\frac{\partial t_a \partial t_b}{\partial x_a \partial x_b}}{\sqrt{\frac{(\partial t_a)^2 (\partial x_b)^2 + (\partial t_b)^2 (\partial x_a)^2}{(\partial x_a \partial x_b)^2}}} \quad (15)$$

$$v_{t(ab)} = \frac{\partial t_a \partial t_b}{\sqrt{(\partial t_a)^2 (\partial x_b)^2 + (\partial t_b)^2 (\partial x_a)^2}} \quad (16)$$

If two particles move in an uniform time velocity,  $\frac{\partial t_a}{\partial x_a} = \frac{\partial t_b}{\partial x_b}$

$$v_{t(ab)} = 0.7071 v_{ta} \quad (17)$$

Also from eq.(16):

$$\int dt = \frac{\partial t_a \partial t_b}{\sqrt{(\partial t_a)^2 (\partial x_b)^2 + (\partial t_b)^2 (\partial x_a)^2}} dx$$

We can assume that  $\partial t_a \partial x_b = \partial t_b \partial x_a = \alpha$ , so the previous equation becomes<sup>[21]</sup>:

$$t = \frac{\partial t_a \partial t_b}{\sqrt{\alpha^2 + \alpha^2}} dx \quad (18)$$

$$t = \frac{\partial t_a \partial t_b}{\sqrt{2} \partial t_a \partial x_b} dx \quad (19)$$

$$t = 0.7071 \frac{\partial t_b}{\partial x_b} \sqrt{(dx_a)^2 + (dx_b)^2}$$

Or

$$t = 0.7071 \sqrt{t_a^2 + t_b^2} \quad (20)$$

**d)The resultant of two time velocities with angle less 90° :**

It is known that the resultant value of two displacement velocities give from the equation:

$$v_x^2 = v_{xa}^2 + v_{xb}^2 - 2v_{xa}v_{xb} \cos\theta$$

By substituting with relation  $v_x = -\frac{1}{v_t}$ , we obtain:

$$\left(-\frac{1}{v_{t(ab)}}\right)^2 = \left(-\frac{1}{v_{ta}}\right)^2 + \left(-\frac{1}{v_{tb}}\right)^2 - 2\left(-\frac{1}{v_{ta}}\right)\left(-\frac{1}{v_{tb}}\right) \cos\theta$$

By simple mathematical treatment, results is:

$$v_{t(ab)} = \frac{v_{ta}v_{tb}}{\sqrt{v_{ta}^2 + v_{tb}^2 - 2v_{ta}v_{tb} \cos\theta}} \quad (21)$$

For ex.:

$$v_{xa} = 2m/s, v_{xb} = 5m/s, v_{ta} = -0.5s/m, v_{tb} = -0.2s/m, \theta = 60^\circ, v_{x(ab)} = 4.3589m/s,$$

$$v_{t(ab)} = -0.2294s/m$$

**e)The resultant of time**

**velocity in three-components:**

To find the time velocity in perpendicular co-ordinates, we use the known vector relation<sup>[2]</sup>:

$$\vec{v}_r = \hat{i} v_x + \hat{j} v_y + \hat{k} v_z \quad (22)$$

Since the displacement velocity vector is equal the negative of the reciprocal time velocity vector or [5]:

$$\vec{v}_r = -\frac{\hat{\lambda}}{v_t}$$

Here  $\lambda$  is unit vector of velocity in three dimension.

Then

$$\frac{\hat{\lambda}}{v_t} = \frac{\hat{i}}{v_{tx}} + \frac{\hat{j}}{v_{ty}} + \frac{\hat{k}}{v_{tz}}$$

The resultant of time velocity vector is given by:

$$\hat{\lambda} v_t^{-1} = \frac{\hat{i} v_{ty} v_{tz} + \hat{j} v_{tx} v_{tz} + \hat{k} v_{tx} v_{ty}}{v_{tx} v_{ty} v_{tz}} \quad (23)$$

The magnitude of the time velocity vector Obtained with using eq.(22):

$$\frac{1}{v_t^2} = \frac{1}{v_{tx}^2} + \frac{1}{v_{ty}^2} + \frac{1}{v_{tz}^2}$$

Results:

$$v_t = \frac{v_{tx} v_{ty} v_{tz}}{\sqrt{v_{ty}^2 v_{tz}^2 + v_{tx}^2 v_{tz}^2 + v_{tx}^2 v_{ty}^2}} \quad (24)$$

Ex:  $v_x = 2m/s, v_y = 4m/s, v_z = 5m/s, v_{tx} = -0.5s/m, v_{ty} = -0.25s/m, v_{tz} = -0.2s/m,$   
 $v_r = 6.7082m/s, v_t = -0.1491s/m$

**f)Relative uniform translational time velocity:**

Let us consider two observers  $o$  and  $o'$  that move, relative to each other, with translational uniform motion. Observer  $o$  sees observer  $o'$  moving with dimension velocity  $v_x$  while  $o'$  sees  $o$  moving with velocity  $-v_x$ , or  $o$  sees  $o'$  moving with time velocity  $v_{tx}$ , while  $o'$  sees  $o$  moving with time velocity  $+v_x$ .

With  $v_x$  as their constant relative dimension velocity, we may write  $\vec{oo'} = \vec{v}_x t$  and  $\vec{v}_x = \hat{u}_x v_x$

By using the reciprocal concept :  $\vec{oo'} = \vec{v}_x t = -\hat{u}_x \frac{t}{v_{tx}}$ , the scalar product of two sides by  $\hat{u}_x$  produce<sup>[3]</sup>:

$$\hat{u}_x \cdot \vec{v}_x t = -\hat{u}_x \cdot \hat{u}_x \frac{t}{v_{tx}}$$

$$v_x = -\frac{1}{v_{tx}}$$

Consider now a particle at point A as shown in Fig.(1), we see that[2] :

$$\vec{r}' = \vec{r} - \vec{v}_x t$$

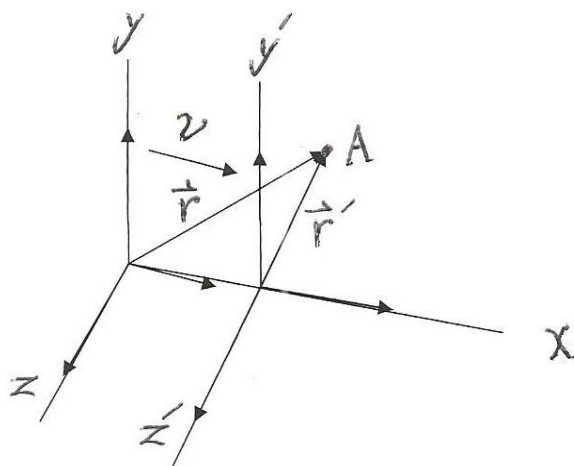


Fig.(1)Frames of reference in uniform relative translational motion[3]



The above vector equation can be separated into its three components, taking into account the fact that  $v_x$  is parallel to axis <sup>[2]</sup>:

$$x' = x - v_x t, y' = y, z' = z, t' = t \quad (26)$$

The last equation that called a Galilean transformation, means that the time measurements are independent of the motion of the observer.

To find the same equations with respect to time velocity, the set equations simply become:

$$x' = x + \frac{t}{v_t}, y' = y, z' = z, t' = t \quad (27)$$

The dimension velocity  $\vec{V}_x$  of A relative to o is defined by:

$$\vec{V}_x = \frac{d\vec{r}}{dt} = \hat{u}_x \frac{dx}{dt} + \hat{u}_y \frac{dy}{dt} + \hat{u}_z \frac{dz}{dt} \quad (28)$$

And the dimension velocity  $\vec{V}'_x$  of A relative to o' is ,

$$\vec{V}'_x = \frac{d\vec{r}'}{dt} = \hat{u}'_x \frac{dx'}{dt} + \hat{u}'_y \frac{dy'}{dt} + \hat{u}'_z \frac{dz'}{dt} \quad (29)$$

When  $t = t'$ . Taking the derivative of eq.(25) relative to time and noting that  $\vec{v}_r$  is constant, so we have:

$$\vec{V}'_r = \vec{V}_r - \vec{v}_r \quad (30)$$

This equation may separate into the three dimension velocity components :

$$\vec{V}'_{x'} = \vec{V}_x - \vec{v}_x, \vec{V}'_{y'} = \vec{V}_y, \vec{V}'_{z'} = \vec{V}_z \quad (31)$$

These equations give the Galilean rule for comparing the dimension velocity of a particle as measured by two observers to relative translational motion.

With respect to time velocity  $\vec{V}_t$  of A relative to o, we can produce, the derivative of time with respect to co-ordinates by substituting in eq.(28):

$$\vec{V}_t = \hat{u}_r \frac{dt}{dr} = \hat{u}_x \frac{dt}{dx} + \hat{u}_y \frac{dt}{dy} + \hat{u}_z \frac{dt}{dz} \quad (32)$$

The time velocity  $\vec{V}_t'$  of A relative to o' depending on eq.(29):

$$\vec{V}_t' = \hat{u}_{r'} \frac{dt}{dr'} = \hat{u}_{x'} \frac{dt}{dx'} + \hat{u}_{y'} \frac{dt}{dy'} + \hat{u}_{z'} \frac{dt}{dz'} \quad (33)$$

At  $t = t'$ , also substitute in eq.(30) by relations:

$$\vec{V}_r' = -\hat{U}_{r'} \frac{1}{V_t'}, \quad \vec{V}_r = -\hat{U}_r \frac{1}{V_t}, \quad \vec{v}_r = -\hat{u}_r \frac{1}{v_t}$$

Results:

$$-\frac{\hat{U}_{r'}}{V_t'} = -\frac{\hat{U}_r}{V_t} + \frac{\hat{u}_r}{v_t} \quad (34)$$

The magnitude can be obtained by using the dimension derivatives in three space, returning to eq.(26) as follow :

$$x' = x - v_x t$$

Derive with respect to time :

$$\frac{dx'}{dt} = \frac{dx}{dt} - v_x$$

$$\frac{dt}{dx'} = \frac{dt}{dx - v_x dt} \quad (35)$$

Divide the right side by  $dt$  we obtain :

$$V_{tx'}' = \frac{V_{tx} v_{tx}}{v_{tx} - V_{tx}} \quad (36)$$

noting that  $V_{tx} = \frac{dt}{dx}, v_{tx} = -\frac{1}{v_x}$

Separate into three time velocity components:

$$V'_{tx} = \frac{V_{tx}v_{tx}}{v_{tx} - V_{tx}}, V'_{ty} = V_{ty}, V'_{tz} = V_{tz} \quad (37)$$

These three equations give another Galilean rule of a particle as measured with time velocity by two observers in relative translational motion.

The dimension acceleration of A relative to  $o$  and  $o'$  is obtained by:

$$\vec{a}_r = \frac{d\vec{V}_r}{dt} \text{ and } \vec{a}'_r = \frac{d\vec{V}'_r}{dt} \text{ respectively using the same } t \text{ in both cases.}$$

From equation (30), noting that  $\frac{dv_x}{dt} = 0$ , because  $v_r$  is constant, however we obtain[2] :

$$\frac{d\vec{V}'_{x'}}{dt} = \frac{d\vec{V}_x}{dt} \text{ or } \vec{a}'_x = \vec{a}_x \quad (38)$$

Which expressed in rectangular coordinates, is  $a'_{x'} = a_x, a'_{y'} = a_y, a'_{z'} = a_z$

In other word, both observers measure the same acceleration, that is, the acceleration of a particle is the same for all observers in uniform relative translational motion, or the dimension acceleration remains invariant when passing from one frame of reference to any other.

The time acceleration that equal the ratio of the change in time velocity to the traveled path( $s/m^2$ ) of a particle A relative to frame of reference  $o$  and  $o'$  or:

$$a_{tx} = \frac{dv_{tx}}{dx} \text{ and } a'_{tx'} = \frac{dV'_{tx'}}{dx'} \text{ respectively}$$

Here we use the same time  $t$ , but different  $x$ .

From eq.(37),differentiae with respect to  $x'$  :

$$\frac{dV'_{tx'}}{dx'} = \frac{(V_{tx} - v_{tx})[V_{tx} \frac{dv_{tx}}{dx} + v_{tx} \frac{dV_{tx}}{dx}] - v_{tx} V_{tx} [\frac{dV_{tx}}{dx} - \frac{dv_{tx}}{dx}]}{(v_{tx} - V_{tx})^2}$$

$$a'_{tx} = \frac{v_{tx}^2 \frac{dV_{tx}}{dx} - V_{tx}^2 \frac{dv_{tx}}{dx}}{(v_{tx} - V_{tx})^2}$$

$$\text{So } a'_{tx} = \frac{V_{tx}^2 a''_{tx} - v_{tx}^2 a_{tx}}{(v_{tx} - V_{tx})^2}, a'_{ty} = \frac{dV_{ty}}{dy}, a'_{tz} = \frac{dV_{tz}}{dz} \quad (39)$$

$$\text{Where } a''_{tx} = \frac{dV_{tx}}{dx}$$

In other words, both observers have the same time acceleration. That is, the time acceleration of a particle is the same for all observers in a uniform relative translational time motion. This result offers us an example of a new physical quantity – the time acceleration of a particle – that appears as the dimensional acceleration to independent of the motion of an observer, in other words, we have found that dimension or time acceleration remains invariant when passing from frame of reference to any other which is in uniform relative translational motion .

**g)Uniform Relative Rotational Time angular velocity:**

Let now consider two observers o and o' rotating relative to each but with no relative translational motion. For simplicity we shall assume that both o and o' are in the same region of space and that each uses a frame of reference attach to itself, but with common region. o' is rotating with dimension angular velocity  $\vec{\omega}_r$ ; o' observes frame XYZ with  $\vec{\omega}_r$ . The position vector  $\vec{r}$  of a particle A referred to XYZ is<sup>[2]</sup> :

$$\vec{r} = \hat{u}_x x + \hat{u}_y y + \hat{u}_z z \quad (40)$$

$$\frac{d\vec{r}}{dt} = \hat{u}_x \frac{dx}{dt} + \hat{u}_y \frac{dy}{dt} + \hat{u}_z \frac{dz}{dt} \quad (41)$$

Similarly, the position vector of A referred to X'Y'Z' is:

$$\vec{r}' = \hat{u}_{x'} x' + \hat{u}_{y'} y' + \hat{u}_{z'} z' \quad (42)$$

The vector  $\vec{r}$  is the same as in eq.(40).

The dimension velocity of A as measured by o' relative to its own frame of reference X'Y'Z' is:

$$\frac{d\vec{r}'}{dt} = \hat{u}_{x'} \frac{dx'}{dt} + \hat{u}_{y'} \frac{dy'}{dt} + \hat{u}_{z'} \frac{dz'}{dt}$$

The time velocity by using the reciprocal dimension base with scalar quantity of eq.(41):

$$\frac{1}{v_{tr}} = \frac{1}{v_{tx}} + \frac{1}{v_{ty}} + \frac{1}{v_{tz}}$$

Or

$$v_{tr} = \frac{v_{tx}v_{ty}v_{tz}}{v_{ty}v_{tz} + v_{tx}v_{tz} + v_{tx}v_{ty}} \quad (43)$$

For ex.  $v_{tx} = -0.2s/m, v_{ty} = -0.5s/m, v_{tz} = 0.25s/m, v_{tr} = -0.09091 s/m, v_r = 11m/s$

In taking the derivative of eq.(42), observer o' has assumed that his frame X'Y'Z' is not rotating, and has therefore considered the unit vectors as constant in direction. However, observer o has the right to say that,

the frame X'Y'Z' is rotating and therefore the unit vectors  $\hat{u}_{x'}, \hat{u}_{y'}, \hat{u}_{z'}$  are not constant in direction, and that in computing the time derivative of Eq.(42) one must write

$$\frac{d\vec{r}'}{dt} = \hat{u}_{x'} \frac{dx'}{dt} + \hat{u}_{y'} \frac{dy'}{dt} + \hat{u}_{z'} \frac{dz'}{dt} + x' \frac{d\hat{u}_{x'}}{dt} + y' \frac{d\hat{u}_{y'}}{dt} + z' \frac{d\hat{u}_{z'}}{dt} \quad (44)$$

Now the endpoints of vectors  $\hat{u}_{x'}, \hat{u}_{y'}, \hat{u}_{z'}$  are (by assumption in uniform circular motion relative to o, with

dimension angular velocity  $\vec{\omega}_r$ . In other words  $\frac{d\hat{u}_{x'}}{dt}$  is the dimension velocity of a point at unit distance

from o and with uniform circular motion with displacement angular velocity  $\vec{\omega}_r$ .

Considering that R remains constant, we obtain :

$$v_s = \frac{ds}{dt} = R \frac{d\theta}{dt}$$

The quantity  $\omega_\theta = \frac{d\theta}{dt}$  is called the dimension angular velocity (rad/s)

Since  $R = r \sin \gamma, v_r = \omega_r r \sin \gamma \Rightarrow \vec{v}_r = \vec{\omega}_r \times \vec{r}$

Therefore

$$\frac{d\hat{u}_{x'}}{dt} = \vec{\omega} \times \hat{u}_{x'}, \quad \frac{d\hat{u}_{y'}}{dt} = \vec{\omega} \times \hat{u}_{y'}, \quad \frac{d\hat{u}_{z'}}{dt} = \vec{\omega} \times \hat{u}_{z'} \quad (45)$$

According to eq.(44) we may write:

$$x' \frac{d\hat{u}_{x'}}{dt} + y' \frac{d\hat{u}_{y'}}{dt} + z' \frac{d\hat{u}_{z'}}{dt} = \vec{\omega} \times \hat{u}_{x'} x' + \vec{\omega} \times \hat{u}_{y'} y' + \vec{\omega} \times \hat{u}_{z'} z' = \vec{\omega}_r \times \vec{r} \quad (46)$$

Introducing this result in eq.(44) and using eq (41) and (46) ,we get:

$$\vec{V}_r = \vec{V}'_r + \vec{\omega}_r \times \vec{r} \quad (47)$$

This expression gives the relation between the dimension velocities  $v_r$  and  $v'_r$ , as recorded by observer  $o$  and  $o'$  in relative distant rotational motion.

To obtain the relation between the dimension accelerations, we proceed in a similar way. The displacement acceleration of A, as measured by  $o$  relative to XYZ is<sup>[2]</sup>

$$\vec{a}_r = \frac{d\vec{V}_r}{dt} = \hat{u}_x \frac{dV_x}{dt} + \hat{u}_y \frac{dV_y}{dt} + \hat{u}_z \frac{dV_z}{dt} \quad (48)$$

The dimension acceleration of A, as measured by  $o'$  relative to X'Y'Z' ,when he again ignores the rotation:

$$\vec{a}'_r = \hat{u}_{x'} \frac{dV_{x'}}{dt} + \hat{u}_{y'} \frac{dV_{y'}}{dt} + \hat{u}_{z'} \frac{dV_{z'}}{dt} \quad (49)$$

When we differentiate eq.(47) with respect to t, remembering that we are assuming that  $\omega_r$  is constant, we obtain:

$$\vec{a}_r = \frac{d\vec{V}_r}{dt} = \frac{d\vec{V}'_r}{dt} + \vec{\omega}'_r \times \frac{d\vec{r}}{dt} \quad (50)$$

$$\vec{a}_r = \frac{d\vec{V}_r}{dt} = \frac{d\vec{V}'_r}{dt} + (\vec{\omega}'_r \times \vec{v}_r)$$

$$\vec{a}_r = \frac{d\vec{V}_r}{dt} = \frac{d\vec{V}'_r}{dt} + \vec{\omega}'_r \times (\vec{\omega}_r \times \vec{r})$$

$$\vec{a}_r = \vec{a}'_r + \vec{\omega}'_r \times (\vec{\omega}_r \times \vec{r}) \quad (51)$$

With respect to angular time velocity and linear time velocity by using the reciprocal velocity  $v_r = -\frac{1}{v_t}$  :

$$\therefore v_t = \frac{dt}{ds} = \frac{dt}{Rd\theta} = \frac{\omega_t}{R}$$

$$\omega_t = v_t r \sin \gamma \quad (52)$$

$$\therefore \vec{\omega}_t = \vec{v}_t \times \vec{r}$$

The direction of  $\vec{\omega}_t$  is the same of direction of  $\vec{\omega}_r$

For ex.  $\omega_r=5\text{rad/s}, r=2\text{m}, \gamma=30^\circ, R=1\text{m}$ , Find  $v_r$ , and verify

Solution:  $v_t=5\text{m/s}, \omega_t=-1/5 \text{ s/rad}$

To find the displacement acceleration follow the equation:

$$\vec{a}_r = \frac{d\vec{v}_r}{dt}, \vec{v}_r = \vec{\omega}_r \times \vec{r} \quad (53)$$

$$\vec{a}_r = \frac{d}{dt}(\vec{\omega}_r \times \vec{r}), \vec{\omega}_r = \text{const.}$$

$$\vec{a}_r = \vec{\omega}_r \times \frac{d\vec{r}}{dt} = (\vec{\omega}_r \times \vec{v}_r) \quad (54)$$

For time acceleration :

$$a_t = \frac{d}{ds} \left( \frac{dt}{ds} \right) = \frac{dv_t}{ds} \quad (55)$$

$$\text{But } v_t = \frac{\omega_t}{r \sin \gamma}$$

Derive with respect to distance becomes:

$$a_t = \frac{d}{ds} \left( \frac{\omega_t}{r \sin \gamma} \right)$$

But  $ds = rd\theta$

$$\therefore a_t = \frac{d}{rd\theta} \left( \frac{\omega_t}{r \sin \gamma} \right)$$

$$\text{Or } a_t = \frac{1}{r^2 \sin \gamma} \left( \frac{d\omega_t}{d\theta} \right) \quad (56)$$

But the time angular acceleration is  $\alpha_t = \frac{d\omega_t}{d\theta}$

$$\text{So } a_t = \frac{\alpha_t}{r^2 \sin \gamma} \quad (57)$$

Where  $\omega_t$  is variable .

For ex.  $r=2\text{m}$ ,  $\gamma=30^\circ$ ,  $\omega_r=5\text{rad/s}$ ,  $v_r=5\text{m/s}$ ,  $a_r=\omega_r v_r \sin \gamma=12.5\text{m/s}^2$   
 $\alpha_t=25\text{s/rad}^2$ ,  $\omega_t=-1/5\text{s/rad}$ .

#### h)The Time Lorentz-Einstein Transformations:

The fourth equation in eq.(26) ( $t = t'$ ) can longer correct, so we may adjust the time as well as the distance if the quotient of the two is to remain the same for observers in relative motion as it does in the case of the dimension velocity of light, that equal to  $2.9979 \times 10^8\text{m/s}$  ,or equal to  $3.335668 \times 10^{-9}\text{s/m}$  or  $3.335668\text{ns/m}$  in time velocity concept ,in other wards, the time interval between two events does not have the same for observers in relative motion.

The new transformation, which is compatible with invariant of the dimension velocity of light, is then<sup>[2]</sup>,

$$x' = \frac{x - v_x t}{\sqrt{1 - \frac{v_x^2}{c_x^2}}}, y' = y, z' = z, t' = \frac{t - \frac{v_x x}{c_x^2}}{\sqrt{1 - \frac{v_x^2}{c_x^2}}} \quad (58)$$

Another new transformation, which is compatible with invariant of the time velocity of light, is then,

$$x' = \frac{x + \frac{t}{v_t}}{\sqrt{1 - \frac{c_t^2}{v_t^2}}}, y' = y, z' = z, t' = \frac{t + \frac{x c_t^2}{v_t}}{\sqrt{1 - \frac{c_t^2}{v_t^2}}} \quad (59)$$



Where  $v_x = -\frac{1}{v_t}, c_x = \frac{1}{c_t}$

The two set equation is called the Lorentz dimension and time transformation respectively. Practically the value of k is equal to one for every dimension and time because  $c_r$  or  $c_t$  is a distance velocity and time velocity very large compared with the great velocities, that is also no difference between the Lorentzian and Galilean transformations in this case.

As we know the Lorentz – or relativistic – transformation must use for very fast particles as the electrons in atoms or particles in cosmic rays.

**i)Transformation of dimension and time velocity**

The dimension and time velocity of A as measured by o has components<sup>[2]</sup> :

$$V_x = \frac{dx}{dt}, V_y = \frac{dy}{dt}, V_z = \frac{dz}{dt}, V_{t_x} = \frac{dt}{dx}, V_{t_y} = \frac{dt}{dy}, V_{t_z} = \frac{dt}{dz} \quad (60)$$

Similarly, the components of distance and time velocities of A as measured by o' are

$$V'_{x'} = \frac{dx'}{dt'}, V'_{y'} = \frac{dy'}{dt'}, V'_{z'} = \frac{dz'}{dt'}, V'_{t'_{x'}} = \frac{dt'}{dx'}, V'_{t'_{y'}} = \frac{dt'}{dy'}, V'_{t'_{z'}} = \frac{dt'}{dz'} \quad (61)$$

Note that we now use  $dt'$  and not  $dt$ , because  $t$  and  $t'$  are not longer the same. Differentiating equations (23) becomes:

$$dx' = k(dx - v_x dt) = k(V_x - v_x)dt, dy' = dy, dz' = dz, dt' = k(dt - v_x \frac{dx}{c_x^2}) = k(1 - \frac{v_x V_x}{c_x^2})dt \quad (62)$$

Where  $k = \frac{1}{\sqrt{1 - \frac{v_x^2}{c_x^2}}}, V_x = \frac{dx}{dt}$

In first and last equations, dx has been replaced by  $V_x dt$ , according to eq.(60) .Therefore dividing the first three of these equations by the fourth we obtain :

$$V'_x = \frac{dx'}{dt'} = \frac{V_x - v_x}{1 - \frac{v_x V_x}{c_x^2}}, V'_y = \frac{dy'}{dt'} = \frac{V_y}{k(1 - \frac{v_x V_y}{c_x^2})}, V'_z = \frac{dz'}{dt'} = \frac{V_z}{k(1 - \frac{v_x V_z}{c_x^2})} \quad (63)$$

$$\text{also } k = \frac{1}{\sqrt{1 - \frac{c_t^2}{v_t^2}}}$$

$$\text{if } v_t = 0.8c_t = 0.8 \times 3 \times 10^8 = 2.4 \times 10^8 \text{ m/s}$$

$$c_t = 0.8 \times v_t = 0.8 \times 0.416625 \times 10^{-9} = 3.33 \times 10^{-9} \text{ s/m}$$

This set of equations give the law for the Lorentz transformation of velocities , that is ,the role for comparing the velocity of a body or measured by two observers in uniform relative translational motion. A gain this reduces to eq.(26) for relative velocities which are very small compared with the velocity of light.

For length contraction and time dilation, the equation becomes :

$$L = \sqrt{1 - \frac{v_x^2}{c_x^2}} L' \quad (64)$$

$$\text{Or } L = \sqrt{1 - \frac{c_t^2}{v_t^2}} L' \quad (65)$$

$$T = \frac{T'}{\sqrt{1 - \frac{v_x^2}{c_x^2}}} = \frac{T'}{\sqrt{1 - \frac{c_t^2}{v_t^2}}} \quad (66)$$

Where  $T'$  is the time interval measured by an observer  $o'$  at rest with respect to the point where the events occurred, and  $T$  is the time interval measured by an observer  $o$  relative to whom the point is in motion when the events occurred. That is, observer  $o$  saw the events occur at two different positions in space. Since

the factor  $\frac{1}{\sqrt{1 - \frac{c_t^2}{v_t^2}}}$  is larger than 1, equation (66) indicates that  $T$  is greater than  $T'$ . Therefore processes

appear to take longer time when they occur in a body in motion relative to the observer than when the body is at rest relative to the observer than when the body is at rest relative to observer ,that is  $T_{motion} > T_{rest}$ .

## Conclusions

Many equations of relative time velocity compatible with relative dimension velocity are produced to describe the translational and rotational relative motion of a body due to Galilean and Lorentz transformations. This is considered an addition equations to complete the concept of relative kinematical motion of a particle. All new equations that obtained from the known give the most meaning of the relative movement of the particle. The dimension movement of particles is not derive from as a general concept so

that give the truth for translation or rotation. The calculating values of the relative dimension or time velocity are in agreement with each other. The idea about converting the two relative velocities to each other gives complementary to the fixed and moving observers with respect to dimension in addition to time. The concluded equations for relative reciprocal velocity and time Lorentz-Einstein transformations are:(3),(10),(14),(20),(21),(24),(32),(34),(37),(39),(43),(52),(57),(59),(64),(65),(66). These are represented the against equations via the dimension equations that can be also used to describe the motion of particle.

## **References**

- [1] Yavorsky B.M. &Pinsky A.A., 1987, “Fundamental of Physics”, Vol. 1, 3<sup>rd</sup>, Printed Mir Pupisher, Moscow,.
- [2] Alonso- Finn, 1980, “Fundamental University Physics” Fifth Printing Addison- Wesely – Company Inc..
- [3] Trag, Seman, 1976,”Theoretical Mechanics” ,Mir Publishers, Moscow, A Short Course, Translated from the Russian by V.Talmy,.
- [4]John L.Synge & Byron A. Griffith, 1981 “Principle of Mechanics”, 3<sup>rd</sup> International Student Ed..
- [5] F.A.Mustafa,2006,”New Formulation of the Reciprocal of Velocity for Particle Motion” Vol.12,No.3, ,J. of Babylon Univ.
- [6] Larry Andrew, Ronald Philips, 2003,” Mathematical Techniques for Engineers and Scientists”, Publication April, Published by SPIE, the International, Society for Optical Engineering,.
- [7] M. Anderson ,2002,”Foundations of Computational Mathematics”, FCM 2002, Minneapolis, 5-14, Classification Problems in Germany, , August.
- [8]Lim Siow Jin, 2002DXN Institute- Wiseman DXN, 213, Lebuhraya S. A. Halim, 05400,Alor Setar, Kedah, Malaysia, 16 June.