Calculation of Drag Coefficients for Geometrical Shapes in Gas Using Numerically Transport Equation Solution

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Abstract

In this work, calculated the drag coefficients of helium gas for geometrical shapes, such as, perpendicular disc, tilted disc and sphere according to the elastic collision by applied the finite difference to solve the numerically transport equation and obtained the electron energy average, this values fed the derivate equations, addition to calculations the electron velocity average and distribution function in term of the electron energy at values, $E/N=(0.12\text{-}30.3)\times10^{-18}$ V. cm^2 and temperature, 300^o Kelvin.

The results obtained appeared the agreement with experimental and theoretical data.

Keyword: drag coefficient, transport equation, geometrical shapes, finite difference, and electron velocity.

1. Introduction

The gas particles, which in the most general case are molecules, either in the normal state or excited or ionized, and free electrons, may perturb the motion of the incoming electron. This perturbation is interpreted as the effect of collisions of the electron with the gas particles. To use the experimental results for the definition of the basic electron collision parameters, in which we are interested, the size of the slab and the composition and density of the gas must be properly chosen. It is clear that the gas density and composition in the experiment have simple to be chosen consistently with the range of values of these same quantities in gas discharge and plasma problems of usual interest the size of the disk-shaped gas sample.

For an elastic collision between an incoming electron and a gas particle of molecular mass M>>m having a much lower kinetic energy, the classical theory of collisions predicts that the electron velocity (modulus) variation is on the average of the order of fraction m/M of the velocity itself [Pauli,2000, Paul,1999, Lachish,1978].

We consider the case of elastic collision to determine the smallest velocity evaluations, the term “elastic collision” means, as in customary in the physics of collisions that involve only the exchange of kinetic energy.

The gas pressure is equal to the momentum delivered to a unit area of a wall, during a unit time [Sitar, 1993].

2.The Boltzmann Equation

Consider an electron swarm in a gas under the action of a continuous Lorentz force field $(E+v\times B)$, where E, B are electric and magnetic fields in general functions of time and position. The number of electrons at time t in the volume dr centered at r in space form and it's velocities in volume dv countered at V in velocity space due to collisions is $f(r,v,t)\,dr\,dv$. The density distribution function f can be computed by solving an integrodifferential equation, which expresses mathematically the property of continuity for the flow of electrons in phase space, it's called Boltzmann equation, which is [Aldo, 1972- V. Guerra, 2001].

$$\frac{\partial f}{\partial t}+v.\nabla_r f-\frac{e}{m}(E+v\times B).\nabla_v f=\overline{B}(r,v,t) \quad (1)$$

Where $\overline{B}$ identifies the Boltzmann term, which provides the electron distribution function rate of change with time due to collisions[N. A. Dyatko,2003], e and m are charge and electron mass respectively. The electron collisions involving a change in the number of interacting particles, such as, ionizing collisions which is called transformation collisions.

3.Solution of Boltzmann Equation

We solved Eq.(1) to obtain the linear equation in a flux divergent form. The result is a finite set of coupled, linear differential equations which state the electrons number density at each energy as a function of time, by feeding the No. of energy mesh points $\leq$ 250 and energy zoning 0.01(eV). From this technique we obtained the transport coefficients as shown in table (1) [Rockwood, 1980].

1- Gas Pressure

A particle moves with a velocity $\underline{V}$ at an angle φ to the perpendicular of a wall as seen in figure(1). It will collide with the wall during a time Δt, if it's distance from the wall is less then $|\underline{V}|\cos\varphi\Delta t$ [Lachish1978, Uri, 2007].

We define the average velocity of particle for three- dimensional, which is:

$$<v>=\int 4\pi v^2 f(v)v\,dv \quad (2)$$

Where: $<v>\neq 0$, since the integration is only over positive values.

$$\int \cos\varphi \sin\varphi\, d\varphi=1,\ (0<\varphi<\pi/2) \quad (3)$$

We can find for momentum ΔM is:

$$\Delta M=n2m(\int 2\pi v^2 f(v)v^2 dv)(\int \cos^2\varphi \sin\varphi\, d\varphi)\Delta t \quad (4)$$

Where: n represents the particle number density, $f(v)$ is the probability that the particle velocity is V, ΔM represents overall momentum, which is the sum of moment a delivered by all the colliding particles that come from all directions of the half sphere, $2mv\cos\varphi$ is the momentum transferred to the wall at particle collision with wall from Eq.(4) we find:

The average for three- dimensional is:

$$< v^2 >= \int 4\pi v^2 f(v) v^2 dv \qquad (5)$$

$$\int \cos^2\varphi \sin\varphi \, d\varphi = 1/3 \qquad (6)$$

Substitute Eqs.(5-6) into Eq.(4) yields:

$$\Delta M = \frac{nm < v^2 >}{3} \Delta t \qquad (7)$$

$$\frac{\Delta M}{\Delta t} = \frac{nm < v^2 >}{3} \qquad (8)$$

Since:

$$\frac{\Delta M}{\Delta t} = P \qquad (9)$$

Where: P represents the pressure

Substitute Eq.(9) into Eq.(8) yields:

$$P = nm < v^2 > /3 \qquad (10)$$

We can find the thermal average $<v>$, $<v^2>$ by using Maxwell-Boltzmann distribution, which is:

$$f(v) = (m/2\pi KT)^{3/2} \exp(-mv^2/2KT) \qquad (11)$$

where: K is the Boltzmann constant, and T is the particle temperature, in unit of Kelvin (^oK)

$$< v >= 4\pi \int f(v) v^3 dv$$

$$= 2\left(\frac{2KT}{\pi m}\right)^{1/2} \qquad (12)$$

$$< v^2 > = 4\pi \int f(v) v^4 dv$$
$$= \frac{3KT}{m} \qquad (13)$$

Substituting Eq.(13) into Eq.(10) yields:

$$P = nm \frac{3KT}{3m}$$
$$= nKT \qquad (14)$$

From the Eq.(14) we can say this equation is the state equation of an ideal gas.

2- Drag of a perpendicular disc

We suppose a flat horizontal disc, of unit area, moving upward at a velocity V_m within a box of an ideal gas [Sears, 1975].

The particles of the gas collide with the disc's front surface will leave it with a higher velocity and particles that collide with the disc's back surface will leave it with a lower velocity, the particle moves with a velocity V_{in} toward a moving disc at a velocity V_m .

From the figure (2), we can deduce the following:

$(|\underline{V}| \cos\varphi + V_m)\Delta t$: refers the particles above the disc will collide with it if their distance from the disc is less.

$(|\underline{V}| \cos\varphi - V_M)\Delta t$: refers the particles below the disc will collide with it if their distance from the disc is less.

$m(|\underline{V}| \cos\varphi + 2V_M)$: refers the momentum since the disc transfers to a particle that collides with it from above.

$m(|\underline{V}| \cos\varphi - 2V_M)$: refers the momentum transfers to a particle that collides with it from below.

From the above, we can say the net momentum transferred to two such particles is zero. Therefore, a net pressure is formed, opposing the disc movement.

The momentum which is transferred above the disc is:

$$\Delta M_1 = [nm \int (|V| \cos\varphi + 2V_M) f(v) (|V| \cos\varphi + V_M) d^3 v] \Delta t \qquad (15)$$

And below it is:

$$\Delta M_2 = [nm\int (|\underline{V}|\cos\varphi - 2V_M) f(v)(|\underline{V}|\cos\varphi - V_M) d^3v]\Delta t \qquad (16)$$

The net momentum transferred above and below is:

$$\Delta M = [6nm\int V\cos\varphi \; f(v) V_M d^3v]\Delta t \qquad (17)$$

From Eq.(17) we can put:

$$d^3v = 2\pi V^2 dV \sin\varphi \, d\varphi \qquad (18)$$

Substitute Eq.(18) into Eq.(17) yields:

$$\Delta M = 12\pi nmV_M (\int V^3 f(v)dv)(\int \cos\varphi \sin\varphi \, d\varphi)\Delta t \qquad (19)$$

But:

$$4\pi\int V^3 f(v)dv = 2(2KT/\pi m)^{\frac{1}{2}} \qquad (20)$$

$$\int \cos\varphi \sin\varphi \, d\varphi = 1, \quad (0 < \varphi < \pi/2) \qquad (21)$$

Substitute Eqs.(20-21) into Eq.(19) the net pressure becomes:

$$P_{ex} = \frac{\Delta M}{\Delta t} = 6nV_M (2kmT/\pi)^{\frac{1}{2}} \qquad (22)$$

We can define the drag coefficient γ by:

$$P_{ex} = \gamma V_M \qquad (23)$$

Substitute Eq.(23) into Eq.(22) yields:

$$\gamma V_M = 6nV_M (2kmT/\pi)^{\frac{1}{2}}$$

$$\gamma = 6n(2mkT/\pi)^{\frac{1}{2}} \qquad (24)$$

The Eq.(24) refers the drag coefficient for a perpendicular disc.

3- Drag of a tilted disc

The velocity of the particle is V_{in} moves at an angle β toward a tilted moving disc. The velocity of the disc V_M as seen in below figure (3).

From the figure (3) we can see the collision of a particle with a tilted moving disc for three cases [Lachish,1978, Philip,1935]:

(a) Velocity of the particle transformation from a static reference to coordinates that move together with the disc.

(b) Velocity of the particle has a mirror reflection in the moving coordinates.

(c) Back transformation to the static reference.

From the Eqs.(22-24) yields:

The drag coefficient γ is:

$$\gamma = \frac{P_{ex}}{V_M}$$
$$= 6n(2mkT/\pi)^{1/2} \cos^2(\alpha) \quad (25)$$

Where α is the tilt angle.

Eq.(25) refers the drag coefficient for a tilted disc.

4- Drag of a Sphere

We can calculated the drag of the sphere by dividing the upper half of its surface into elements [Lachish,1978, Philip,1935] as shown in fig.(4).

From this could be say:

R refers the sphere radius, φ is the elevation angle in the front look, and θ is the rotation angle from above, and $r = R\sin\varphi$.

The contribution of a surface element is $r\,d\theta\,R\,d\varphi$ to the drag F is [N. A. Dyatko, 2003]:

$$r = R\sin\varphi \quad (26)$$

$$dF = \mathrm{P}_{ex} rd\pi R\cos^2\varphi d\varphi \quad (27)$$

by using Eq.(22) and substitute Eq.(26) into Eq.(27) yields:

$$dF = 6n(2mkT/\pi)^{1/2} V_M d\theta R^2 \sin\varphi\cos^2\varphi d\varphi \quad (28)$$

The integration over the azimuth angle θ is 2π, and the integration over the elevation angle φ, between 0 and $\pi/2$ is 1/3, therefore the drag force is:

$$F = P_{ex}A = \gamma V_M A = 4n(2mkT/\pi)^{1/2} V_M (\pi R^2) \tag{29}$$

Where:

$$A = \pi R^2 \text{ (is the sphere cross section)} \tag{30}$$

Substitute Eq.(30) into Eq.(29) yields: The drag coefficient is:

$$\gamma = 4n(2mkT/\pi)^{\frac{1}{2}} \tag{31}$$

The equation (31) represents the drag coefficient for a sphere.

5.Formulation of the Problem

The distribution function f^o becomes Maxwellian and corresponds to an electron average energy, <u> [Aldo, 1972]:

$$<u> = \frac{2\pi m}{n_e}\int_0^\infty f^o v^1 dv = \frac{3}{2}kT_g \tag{32}$$

And

$$u = \frac{1}{2}mv^2 \tag{33}$$

Where m, n_e, v, k, T_g and u are represent electron mass, electron number density (n_e = 2.5×10^{19}cm^{-3}), electron speed, Boltzmann constant, gas temperature and electron energy respectively. Substitute Eqs.(32-33) into Eqs.(11-14) respectively, Eq.(24-25) and Eq.(31) become:

$$f(u) = \left(\frac{3m}{4\pi <u>}\right)^{\frac{3}{2}} \exp\left(-\frac{3u}{2<u>}\right) \tag{34}$$

$$<v> = 2(2kT/\pi m)^{\frac{1}{2}} = 2\left(\frac{4<u>}{3\pi m}\right)^{\frac{1}{2}} \tag{35}$$

$$<v^2> = \frac{3kT}{m} = \frac{3\times 2<u>/3}{m} = \frac{6<u>}{3\ \ m} \tag{36}$$

$$P_{state\ eq.} = nkT = \frac{2}{3}n<u> \tag{37}$$

$$\gamma_{perp.} = 6n(2mkT/\pi)^{\frac{1}{2}} = 6n\left(\frac{4m<u>}{3\quad\pi}\right)^{\frac{1}{2}} \tag{38}$$

$$\gamma_{tilt.} = 6n(2mkT/\pi)^{\frac{1}{2}}\cos^2(\alpha)$$

$$= 6n\left(\frac{4m<u>}{3\quad\pi}\right)^{\frac{1}{2}}\cos^2(\alpha) \tag{39}$$

Where:

$\alpha = 0^o - 90^o$

$$\gamma_{sphere.} = 4n(2mkT/\pi)^{\frac{1}{2}} = 4n\left(\frac{4m<u>}{3\quad\pi}\right)^{\frac{1}{2}} \tag{40}$$

Substitute Eq.(36) into Eq.(10) becomes:

$$\mathrm{P}_{net} = nm<v^2>/3 = \frac{6nm<u>}{9\quad m} = \frac{2n<u>}{3} \tag{41}$$

Where n_e=2.51×10^{19}cm^{-3}.

We constructed a computer program to calculate the above physical equations according to fig.(15).

6.Result & discussion

In this job, we study and analysis of the drag coefficient for the rare gas which it's the helium gas, where are important role play in the industrial applications, such as, which used by artists for special purpose lighting.

Fig (5) represents the electrons distribution function as a function of the electrons speed, since the electron distribution refers the exponential function with its speed increasing according to the equations:

Log(Y)= B×log(x) +A, Log(Y)= —3.00037×log(x) +63.8196, Y= pow (x, —3.00037)×5.20572×10^{27} [14,15].

Fig (6) it was a similar to the fig(1) but as a function of the electron average energy <u>, i.e., the distribution function is a dependence, which it's not depend on the speed or energy of the particle as show in equations:

Log(Y) = B×log(x) +A, Log(Y)= —1.50001×log(x) +(—30.6265), Y= pow (x, —1.50001) ×5.00111×10^{-14}.

Fig (7-8) was shown the three–dimensional particles average velocity is increasing with Particle average energy according to equation:

Log(Y)= B×log(x) +A, Log(Y)= 0.4999×log(x) +31.3962, Y= pow(x, 0.49999) ×4.3171×10^{13}, as in fig(7) but in fig(8) the relation between the particle average velocity square and the average energy is straight line according to the equation:

Linear, Y=B× x+ A, Y= 2.19566×10^{27} × x+ (—5.11673×10^{22}), i.e. the particle acquired the energy from the elective field.

Fig(9) refers the particle net momentum transferred above and below the disc, ΔM_1, Eq.(15) and ΔM_2, Eq.(16) respectively for drag of a perpendicular disc as a function of the average energy which acquired from the electric field according to the equation:

Linear, Y= B× x+ A, Y= 1.67328×10^{19}× x+ 2.32771×10^{14}. Since it's straight line.

Fig (10) represents the state equation of an ideal gas. This pressure is a function of the gas average energy instead of KT. This relation is a straight line according to the equation:

Linear, Y=B× x+ A, Y=1.67328×10^{19} × x+ 2.32771×10^{14}.

Fig (11) refers the drag coefficient of a perpendicular disc as a function of the particle average energy as seen in Eq.(38). At <u>=(0.051-0.102) eV, the drag coefficient increases high but after these energies , the drag increases slow. We can say from the above, when the particle average energy increases the drag coefficient is very slow according to the equation:

Power, log(Y)= B× log(x)+ A, Log(Y)= 0.500003* log(x)+ 14.901, Y=pow(x, 0.500003) ×2.96102×10^{6}.

Fig(12-13) show the drag coefficient of a titled disc as a function of the particle average energy, Eq.(39) and a tilt angle (α). From the figures, when the average energy is increasing, the drag coefficient is slow. We could show at the tilt angle $\alpha = 90^0$ the drag coefficient equal zero, but at $\alpha = 45^0$, the drag is increasing and so on as shown in equations:

at $\alpha = 0^0$, log(Y)= 0.500003×log(x)+ 14.901, Y=pow (x, 0.500003) ×2.96102 ×10^{6}

at $\alpha = 45^0$, log(Y)=0.499947×log(x) +14.2078, Y=pow(x , 0.499947) ×1.48042 ×10^{6}

at $\alpha = 90^0$, linear, Y=B× x+A, Y= 7.56418×10^{-27} × x +2.83604×10^{-27}

From the above we can say when a tilt angle go to the large, the drag tend to the small values and opposite correct. But when the particle average energy, <u> is increasing the drag coefficient go to the zero as in fig (13).

Fig (14) represents the drag coefficient of a sphere as a function of the practice energy, <u> Eq.(40). When the average energy <u> is increasing, the drag coefficient tend to the slowing according to the equations:

Power, log (Y)=B×log(x)+A, log(Y)=0.499964×log(x)+14.4955, Y=pow(x, 0.499964) ×1.97307×10^{6}

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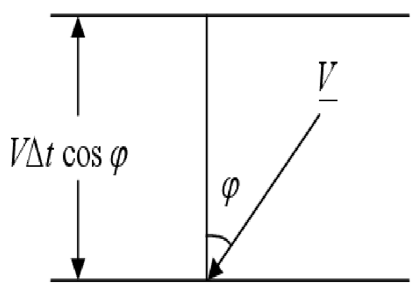
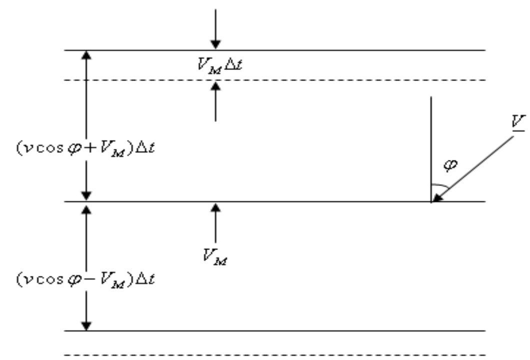
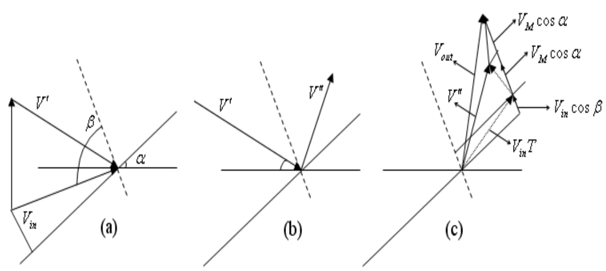


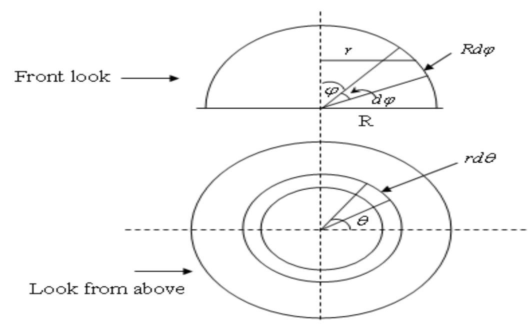
Fig(1): Elastic collision of a particle with a wall



Fig(2): Elastic collision of a particles with a moving disc



Fig(3): Collision of a particle with a tilted disc



Fig(4): The sphere parameters and coordinates. The upper figure is a front look and the lower figure is a look from above. The sphere moves upward.

Table (1): The calculated transport parameter for pure Helium gas

$E/N\times10^{-18}$ ($V.cm^2$)	<u> (eV)
0.12	0.051
0.18	0.052
0.45	0.062
1.21	0.102
1.82	0.136
4.55	0.283
12.14	0.677
18.2	0.989
24.3	1.312
30.3	1.635

Table (2): The calculated physical quantities for Helium gas

$f(v)$ $(eV)^{-3/2}$	$f(u)$ $(eV)^{-3/2}$	$<v>$ $(cm\ sec^{-1})$	$P_{net.}$ (Torr)	P_{state} (Torr)	$\gamma_{perp.}$	γ_{sphere}
4.342×10^{-12}	4.342×10^{-12}	9.749×10^{12}	8.534×10^{17}	8.534×10^{17}	6.687×10^{5}	4.458×10^{5}
4.218×10^{-12}	4.218×10^{-12}	9.844×10^{12}	8.701×10^{17}	8.701×10^{17}	6.752×10^{5}	4.502×10^{5}
3.240×10^{-12}	3.240×10^{-12}	1.075×10^{13}	1.037×10^{18}	1.037×10^{18}	7.373×10^{5}	4.915×10^{5}
1.535×10^{-12}	1.535×10^{-12}	1.379×10^{13}	1.707×10^{18}	1.707×10^{18}	9.457×10^{5}	6.305×10^{5}
9.972×10^{-13}	9.972×10^{-13}	1.592×10^{13}	2.276×10^{18}	2.276×10^{18}	1.092×10^{6}	7.280×10^{5}
3.322×10^{-13}	3.322×10^{-13}	2.297×10^{13}	4.736×10^{18}	4.736×10^{18}	1.575×10^{6}	1.050×10^{6}
8.978×10^{-12}	8.978×10^{-14}	3.552×10^{13}	1.133×10^{19}	1.133×10^{19}	2.436×10^{6}	1.624×10^{6}
5.085×10^{-14}	5.085×10^{-14}	4.293×10^{13}	1.655×10^{19}	1.655×10^{19}	2.945×10^{6}	1.963×10^{6}
3.328×10^{-14}	3.328×10^{-14}	4.945×10^{13}	2.195×10^{19}	2.195×10^{19}	3.392×10^{6}	2.261×10^{6}
2.392×10^{-14}	2.392×10^{-14}	5.520×10^{13}	2.736×10^{19}	2.736×10^{19}	3.786×10^{6}	2.524×10^{6}

Table (3): The calculated drag coefficient of a tilted disc for Helium gas

$\gamma t(u,0)$	$\gamma t(u,45)$	$\gamma t(u,90)$	$\gamma t(u_0,\alpha)$	$\gamma t(u_5,\alpha)$	$\gamma t(u_9,\alpha)$
6.687×10^{5}	3.344×10^{5}	2.507×10^{-27}	6.687×10^{5}	1.575×10^{6}	3.786×10^{6}
6.752×10^{5}	3.376×10^{5}	2.532×10^{-27}	6.485×10^{5}	1.528×10^{6}	3.672×10^{6}
7.373×10^{5}	3.687×10^{5}	2.764×10^{-27}	5.905×10^{5}	1.391×10^{6}	3.343×10^{6}
9.457×10^{5}	4.729×10^{5}	3.546×10^{-27}	5.015×10^{5}	1.181×10^{6}	2.840×10^{6}
1.092×10^{6}	5.460×10^{5}	4.094×10^{-27}	3.924×10^{5}	9.244×10^{5}	2.222×10^{6}
1.575×10^{6}	7.876×10^{5}	5.906×10^{-27}	2.763×10^{5}	6.509×10^{5}	1.564×10^{6}
2.436×10^{6}	1.218×10^{6}	9.134×10^{-27}	1.672×10^{5}	3.938×10^{5}	9.466×10^{5}
2.945×10^{6}	1.472×10^{6}	1.104×10^{-26}	7.822×10^{4}	1.843×10^{5}	4.429×10^{5}
3.392×10^{6}	1.696×10^{6}	1.272×10^{-26}	2.016×10^{4}	4.750×10^{4}	1.142×10^{5}
3.786×10^{6}	1.893×10^{6}	1.420×10^{-26}	2.507×10^{-27}	5.906×10^{-27}	1.420×10^{-26}

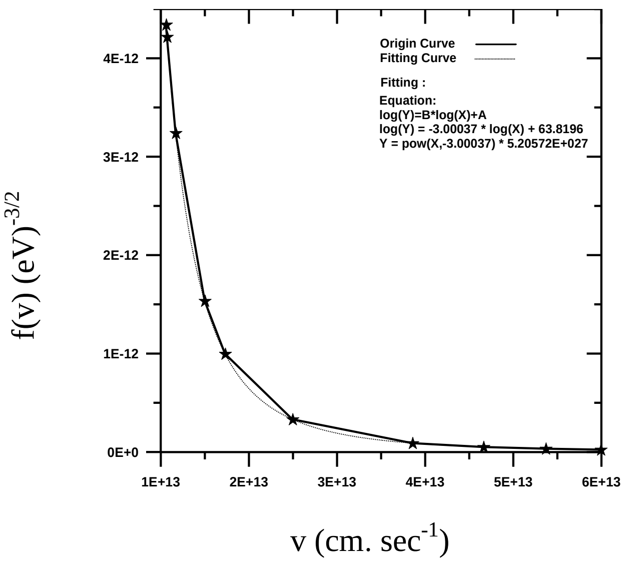


Fig.(5): The particle distribution function as a function of the particle speed in a pure Helium gas.

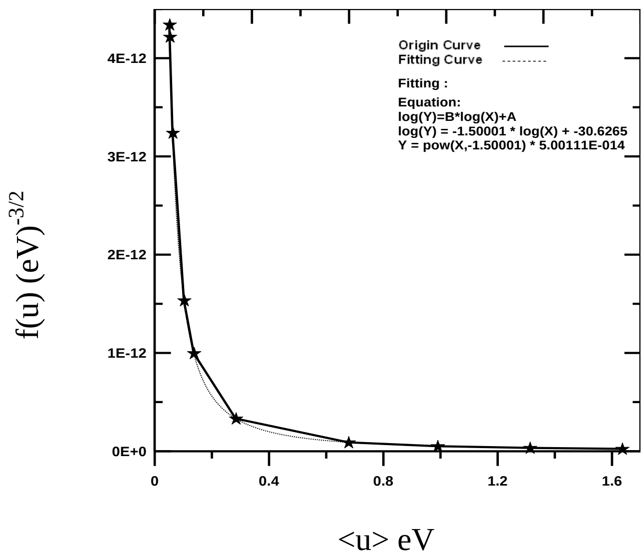


Fig.(6): The particle distribution function as a function of the particle average energy, <u> in Helium gas.

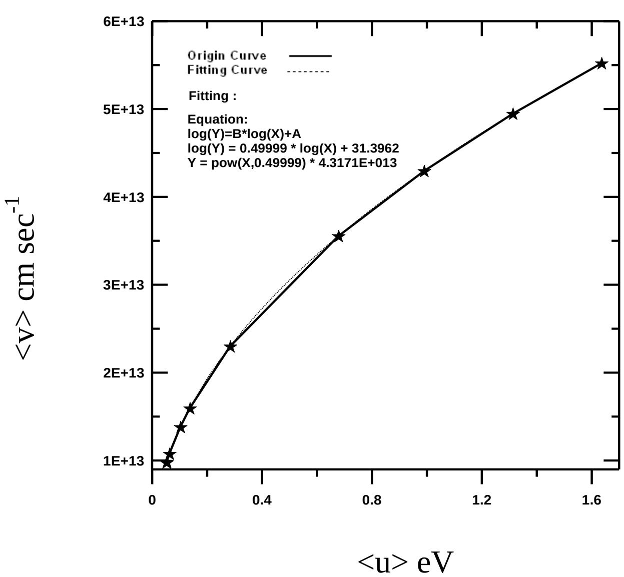


Fig.(7): The average velocity of the particles as a function of the particle average energy, <u> in Helium gas.

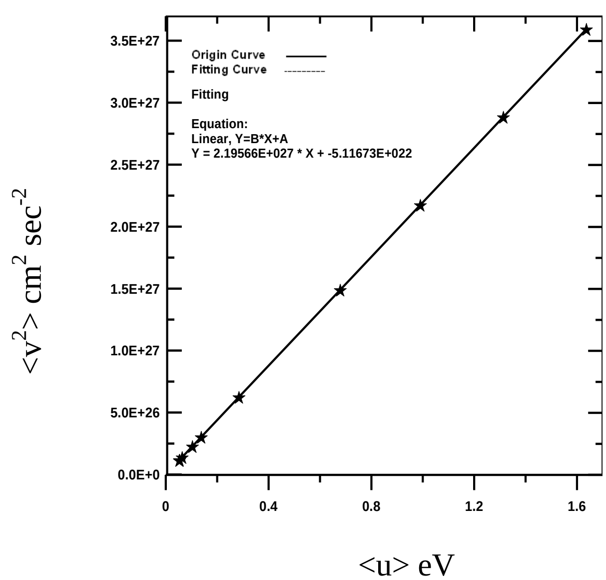


Fig.(8): The particle average velocity squire as a function of the particle average energy, <u> in Helium gas.

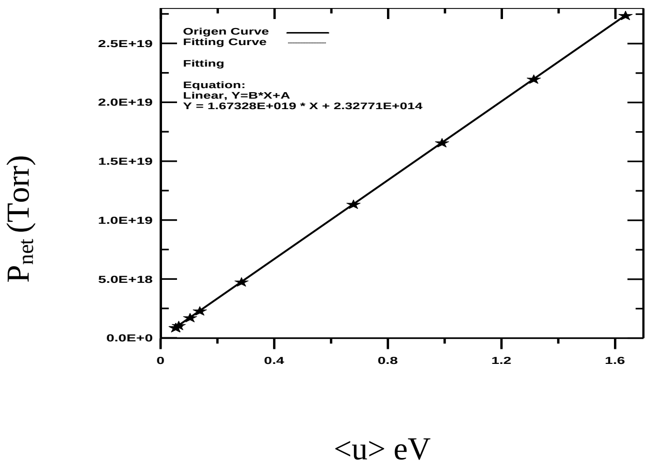


Fig.(9): The particle net momentum as a function of the particle average energy, <u> in pure Helium gas.

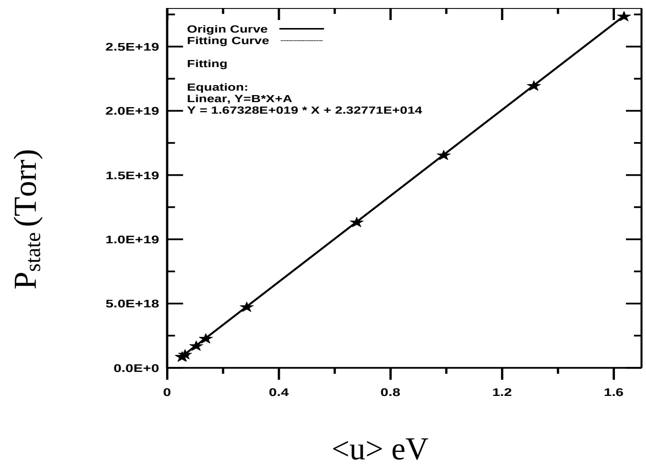


Fig.(10): The particle pressure as a function of the particle average energy, <u> in pure Helium gas.

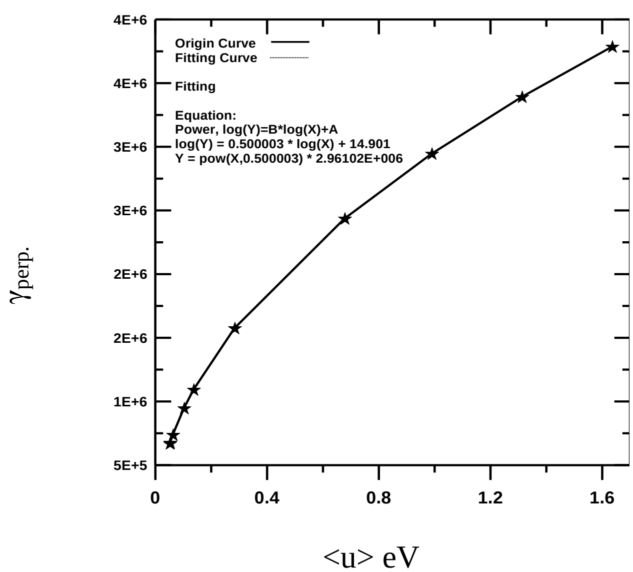


Fig.(11): The drag coefficient of a perpendicular disc as a function of the particle average energy, <u> in pure Helium gas.

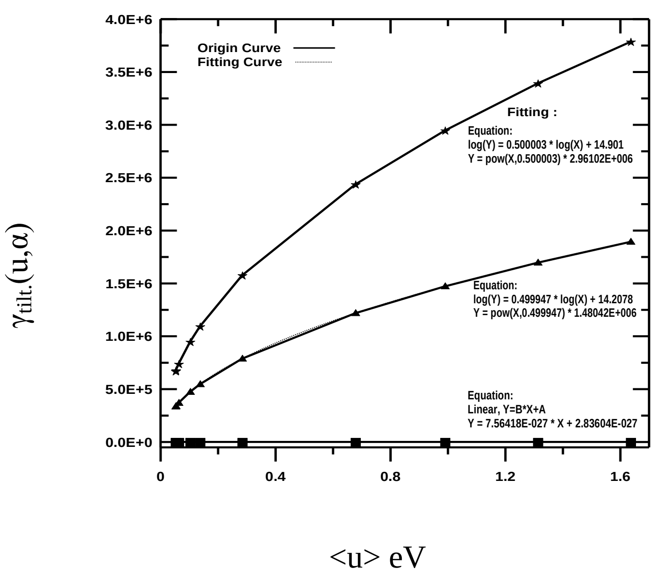
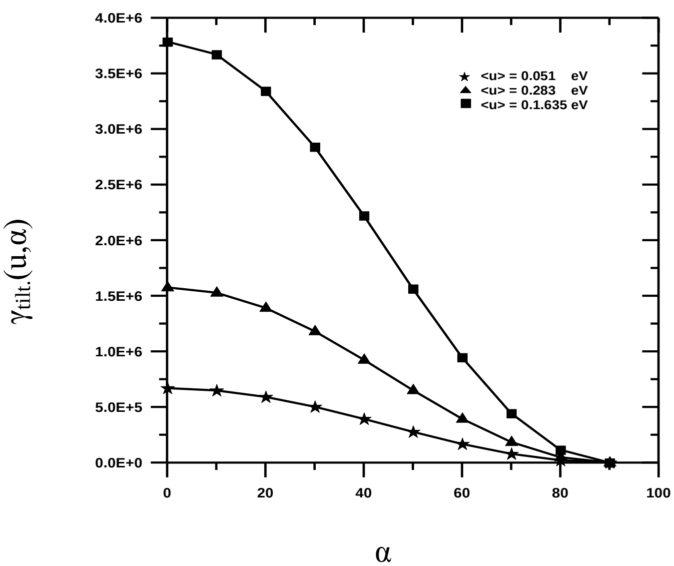


Fig.(12): The drag coefficient of a tilted disc as a function of the particle average energy, <u> at a different tilt angle(α) in pure Helium gas.



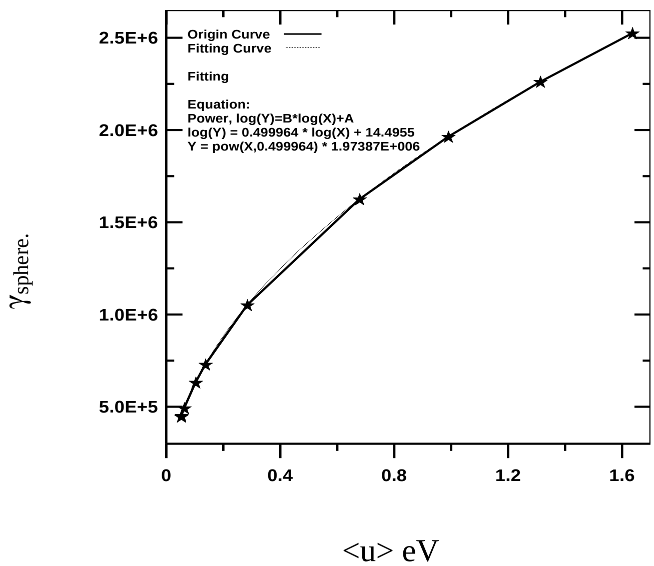


Fig.(13): The drag coefficient of a tilted disc as a function of the tilt, α at a different energies in pure Helium gas.

Fig.(14): The drag coefficient of a sphere as a function of the particle average energy, <u> in pure Helium gas.

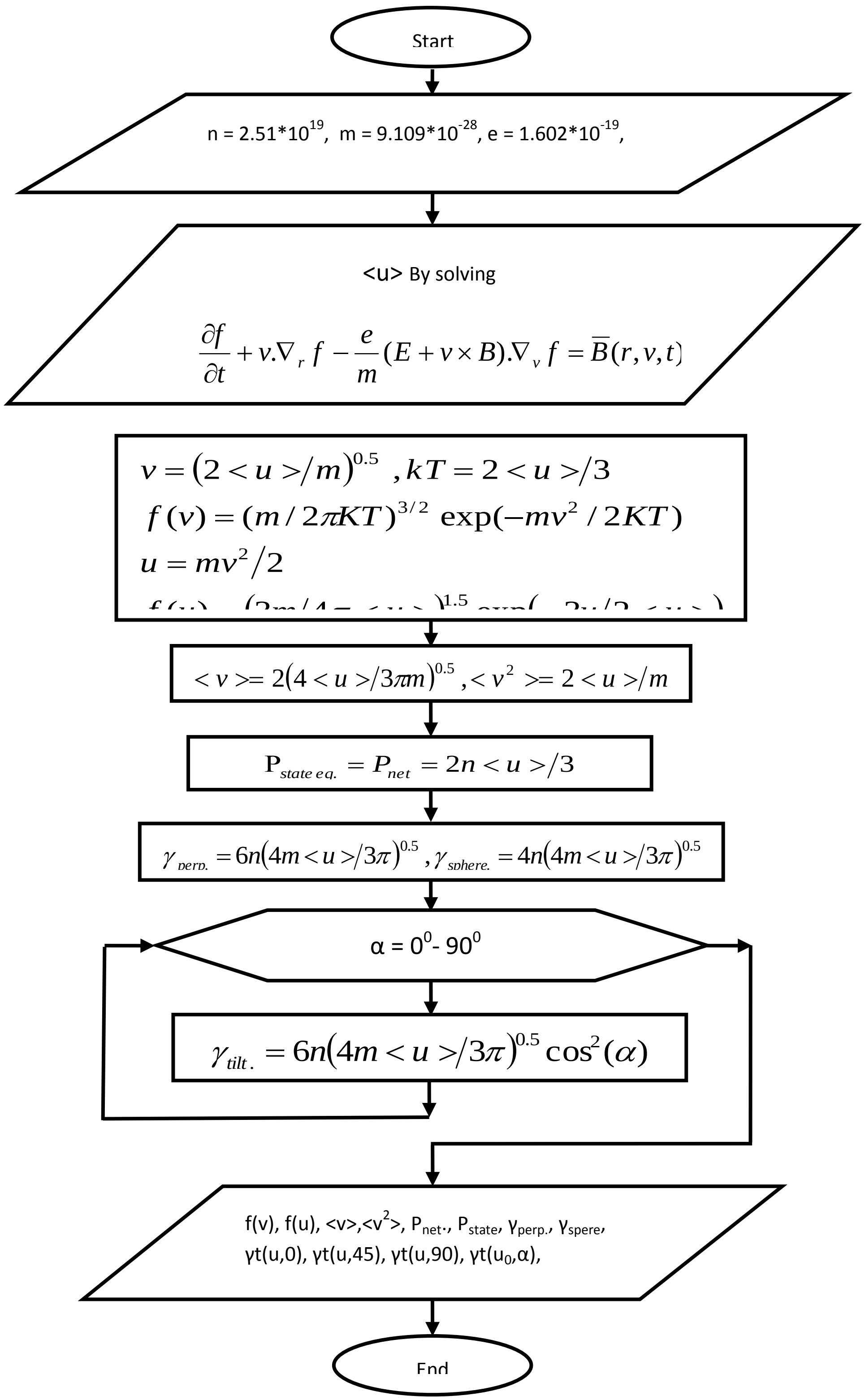


Fig.(15): Diagram Flow Chart