Theoretically Investigation of Experimental Values for the Behaviour of Electrons in Ionosphere

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Abstract

In this job, the theoretical investigation for experimental values of the motions of free electrons in air and nitrogen gas. The following parameters of the electronic motion were determined as functions of the ratio E/P of the applied electric field E to the gas pressure P, Townsend's energy factor K_T, the drift velocity W, the mean free path at unit pressure L and the mean proportion η of it's energy lost in collisions with gas molecules. The study appeared a good agreement between the theoretical values and the experimental values as shown in the figures.

Keyword: Electron energy distribution, Boltzmann transport equation, Maxwell speed distribution.

1. Introduction

An account of a theoretical and experimental investigation of the slow electron motion in the air using a diffusion method for Townsend, this fact that when electrons enter the diffusion chamber through a small hole in the upper electrode and move in an uniform electric field in a steady state of motion through the gas the lower electrode, the hole behaves as a doublet source and the mathematical expression for the distribution of the current over the receiving electrode is relatively simple, from this, it's to change the design of an apparatus to suit the experimental condition without becoming committed to a formidable program of computation [Crompton,1952, Matsuura,2008].

In this work obtained calculation of the losses of energy of slow electrons in collisions with molecules of air and N_2 gas, in order to use them in parallel investigations of the properties of the E-region of the ionosphere by means of the phenomenon of the interaction of radio waves [Crompton, 1953].

We shall consider two only of the possible laws of distribution, Maxwell's law for which

$$K_T = K_1 \qquad (1)$$

And Druyvesteyn's law for which

$$K_T = K_1 / 1.14 \qquad (2)$$

2. Calculations

2.1. Determination of the drift velocity W

It is necessary to measure the drift velocity, W for electrons by observing the defluxion of the stream of electrons in a magnetic field, H.

An elementary application of theory of the Hall Effect gives the following formula for W [Aldo, 1972]:

$$W = \frac{E}{H}\tan\theta = \frac{E}{H}\frac{b}{h} \qquad (3)$$

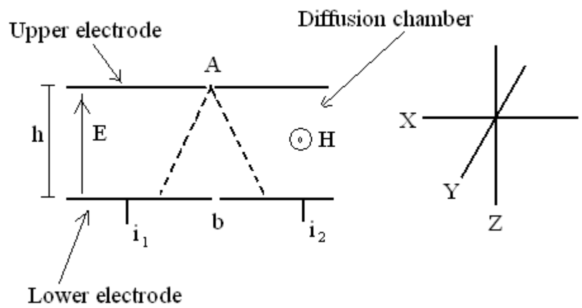

Fig.(1): Schematic diagram of diffusion apparatus

Whereas,

$$\tan\theta = \frac{W_x}{W_z} \tag{4}$$

$$= \frac{\omega \overline{T^2}}{2\overline{T}} \tag{5}$$

Whereas

$$\omega = \frac{H\,e}{m} \tag{6}$$

$$T = \ell / U \tag{7}$$

where, E represents the electric field in unit of (V/cm) and H in Orested, A is an aperture, h is a depth of the chamber, b is the distance between the two currents i_1, i_2, W_x is the component of the drift velocity at right angles to the E and H, W_z is the component parallel to E. ω is a cyclotron frequency, e and m are electronic charge and mass respectively, ℓ is mean free path and U is the electrons speed.

The drift velocity W of the centre of mass of a group of electrons moving through a gas in a constant and uniform electric field E is:

$$W = \frac{2}{3}\frac{E\,e}{m}\overline{\left(\frac{\ell}{U}\right)} \tag{8}$$

Substitute Eq.(7) into Eq.(8) yields:

$$W = \frac{2}{3}\frac{E\,e}{m}\overline{T} \tag{9}$$

$$3mW = 2E\,e\,\overline{T}$$

$$\overline{T} = \frac{3mW}{2E\,e} \tag{10}$$

Substitute Eq.(6) into Eq.(5) yields:

$$\tan\theta = \frac{H\,(e/m)\,\overline{T^2}}{2\quad \overline{T}}$$

$$= \frac{1}{2}\frac{H\,e}{m}\frac{\overline{T^2}}{\overline{T}}$$

$$= \frac{1}{2}\frac{H\,e}{m}\overline{T}\frac{\overline{T^2}}{\left(\overline{T}\right)^2} \qquad (11)$$

Substituting Eq. (10) into Eq. (11) yields:

$$\tan\theta = \frac{1}{2}\frac{H\,e}{m}\frac{3mW}{2E\,e}\left[\frac{\overline{(U^{-2})}}{\overline{(U^{-1})}^2}\right]$$

$$= \frac{3}{4}\frac{H\,W}{E}\left[\frac{\overline{(U^{-2})}}{\overline{(U^{-1})}^2}\right]$$

or

$$W = \frac{4}{3}\frac{E\ \tan\theta}{H\left[\frac{\overline{(U^{-2})}}{\overline{(U^{-1})}^2}\right]}$$

$$= \frac{4}{3}\left[\frac{\overline{(U^{-1})}^2}{\overline{(U^{-2})}}\right]\frac{E}{H}\tan\theta \qquad (12)$$

From Ref. [Huxley, 1949] could be finding:

$$\left[\frac{\overline{(U^{-1})}^2}{\overline{(U^{-2})}}\right] = 0.639 \quad \text{….for Maxwell} \quad (13)$$

$$\left[\frac{\overline{(U^{-1})}^2}{\overline{(U^{-2})}}\right] = 0.707 \quad \text{….for Druyvesteyn} \quad (14)$$

Substitute eqs. (13, 14) into Eq. (12) yields:

$$W = 0.851\frac{E}{H}\tan\theta \qquad \text{(Maxwell)} \quad (15)$$

$$W = 0.9426\frac{E}{H}\tan\theta \qquad \text{(Druyvesteyn)} \quad (16)$$

Whereas [Aldo, 1972]:

$$\tan\theta = \frac{e\,H}{m\nu_m} \qquad (17)$$

$$\theta = \tan^{-1}\left(\frac{e}{m}\frac{H}{\nu_m}\right) \qquad (18)$$

$$H = \frac{E}{V_d}\left(K_1 - 1\right)^{1/2} \qquad (19)$$

$$K_1 = \frac{e}{K\,T_g}\frac{D}{\mu} \qquad (20)$$

Where, ν_m represents the momentum transfer collision frequency, K_1 refers Townsend's energy factor, V_d refers the electron drift velocity, K is the Boltzmann constant =1.3805×10^{-23} J ºK^{-1}, T_g is the gas temperature in Kelvin=300ºK and D/μ refers the electron characteristic energy in unit of (eV).

2.2. Distribution of the velocities

K_T Proves (at a fixed temperature) to be a function of the ratio of the electric field strength E to the gas pressure P. The root mean-square velocity $\sqrt{(\overline{U^2})}$ in unit of cm/sec is obtained [Crompton,1953]:

$$\left(\overline{U^2}\right)^{\frac{1}{2}} = 1.15\times10^7 \left(K_T\right)^{\frac{1}{2}} \quad \text{(Maxwell) (21)} \quad (K_1 = K_T)$$

$$\left(\overline{U^2}\right)^{\frac{1}{2}} = 1.08\times10^7 \left(K_1\right)^{\frac{1}{2}} \quad \text{(Druyvesteyn)} \quad (22)$$

$$\overline{U} = 1.06\times10^7 \left(K_1\right)^{\frac{1}{2}} \quad \text{(Maxwell)} \quad (23)$$

$$\overline{U} = 1.02\times10^7 \left(K_1\right)^{\frac{1}{2}} \quad \text{(Druyvesteyn)} \quad (24)$$

2.3. The proportion η of its energy lost in the mean by an electron per collision

The mean proportion η of its energy lost of the average by an electron in a collision with a molecule, i.e., when electron drifts with velocity W through gas, it gains power (ω= EeW) from electric field strength.

This power dissipated in a collision with the molecules in case of steady state motion, i.e., (time equals zero). Overall the electron lost more energy in some collision than gained from the field.

The electron agitation mean energy of motion in case of steady state is Q:

$$Q = \frac{1}{2} m\overline{U^2} \quad (25)$$

And the loss of energy per collision is ηQ. It follows that:

$$\left(\frac{\overline{U}}{L}\right)(\eta Q) = E\,eW \quad (26)$$

Substitute Eq. (25) into Eq. (26) yields:

$$\eta = \frac{E\,eWL}{\frac{1}{2} m\overline{U^2}\,\overline{U}} = \frac{2E\,eWL}{m\overline{U^2}\,\overline{U}}$$

$$\eta = \frac{3W^2}{\overline{(U^{-1})}\,\overline{U}\,\overline{U^2}} \quad (27)$$

where

$$\left(\overline{U^2}\right)^{1/2} = 1.15\times10^7 \left(K_T\right)^{1/2}, cm\sec^{-1} \quad (28)$$

By squaring two sides yields:

$$\overline{U^2} = (1.15\times10^7)^2 \left(K_T\right) \quad (29)$$

Since[Crompton,1952]:

$$K_T = K_1 / A \quad (30)$$

Where

$$A = \frac{3}{2}\left[\frac{\overline{U}}{\overline{U^2}\,\overline{(U^{-1})}}\right] \quad (31)$$

From Eq. (30) yields:

$$K_T = \frac{2}{3}\left[\frac{\overline{U^2}\,\overline{(U^{-1})}}{\overline{U}}\right]K_1 \quad (32)$$

Substitute Eq. (32) into Eq. (29) yields:

$$\overline{U^2} = \left(1.15\times10^7\right)^2 \frac{2}{3}\left[\frac{\overline{U^2}\,\overline{(U^{-1})}}{\overline{U}}\right]K_1 \quad (33)$$

$$= 0.882\times10^{14}\,\frac{\overline{U^2}\,\overline{(U^{-1})}}{\overline{U}}\,K_1 \quad (34)$$

Substitute Eq. (34) into Eq. (27) yields:

$$\eta = \frac{3W^2}{\left[\overline{(U^{-1})}\,\overline{U}\right]\times 0.882\times10^{14}\,\frac{\overline{U^2}\,\overline{(U^{-1})}}{\overline{U}}\,K_1}$$

$$\eta = \frac{3.409\times10^{-14}W^2}{\overline{(U^{-1})}^2\,\overline{U^2}\,K_1} \quad (35)$$

According to the Eqs.(13, 14) could be find:

$$\eta = 1.79\times10^{-14}\,\frac{W^2}{K_1} \quad \text{(Maxwell)} \quad (36)$$

$$\eta = 1.68\times10^{-14}\,\frac{W^2}{K_1} \quad \text{(Druyvesteyn)} \quad (37)$$

But when using the theory of the interaction of radio waves in the ionosphere, that η gives by the form:

$$\eta = G\left(1 - \frac{1}{K_T}\right) \quad (38)$$

at, $E/P \langle 2$

$G = 1.3\times10^{-3}$ in case of Druyvesteyn law.

2.4. Mean free path at unit pressure, L

We can write the electron mean free path at unit pressure (1mm Hg) if the experimental energy factor K_1 and the drift velocity W are known, which is [Huxley, 1949]:

$$L = \frac{3}{2}\frac{m}{e}\frac{W}{E/P}\frac{1}{\overline{(U^{-1})}} \quad (39)$$

Where e, m, U and P are respectively the electronic charge, electron mass, velocity of agitation and the pressure of the gas.

Consider Nielsen & Bradbury's values for W, if supposed W to be such an experimental quantity, from the above could be write Eq. (39) in the form:

$$L = \frac{3}{2}\frac{m}{e}\frac{W}{(E/P)}\left(\overline{U^2}\right)^{\frac{1}{2}}\left[\frac{1}{\overline{(U^{-1})}\,\overline{(U^2)}^{1/2}}\right] \quad (40)$$

Substitute eqs.(28, 32) into Eq.(40) yields:

$$L = 7.96 \times 10^{-9} \frac{W}{(E/P)} (K_1)^{1/2} \left[\overline{U}\,\overline{U^{-1}}\right]^{-1/2} \quad (41)$$

It follows from Eqs.(4-5) that

$$\tan\theta = \frac{\omega h}{2P}\left[\frac{\overline{U^{-2}}}{\overline{U^{-1}}}\right]$$

By using eqs. (21-22, 31), that

$$L = 1.06 \frac{E}{H} \frac{\tan\theta\sqrt{K_1}}{(E/P)} \left[\frac{\left(\overline{U^{-1}}\right)^{\frac{1}{2}}}{\left(\overline{U}\right)^{\frac{1}{2}} \overline{U^{-2}}}\right] \qquad (42)$$

According to the Eqs. (13, 14) obtained:

$$L = 0.598 \frac{E}{H} \frac{\tan\theta}{(E/P)} \sqrt{(K_1)} \text{ (Maxwell)} \qquad (43)$$

$$L = 0.689 \frac{E}{H} \frac{\tan\theta}{(E/P)} \sqrt{(K_1)} \text{ (Druyvesteyn)} \quad (44)$$

eqs.(43-44) are express L in terms of experimental quantities.

2.5. Transport coefficient calculation

The distribution function $f(\nu)$ represents by the first two terms alone of its expansion in spherical harmonics in velocity and that the spherically symmetric term f° is predominant since collisions tend to disorder any directional motion of the electrons. Whether, the Boltzmann equation is [Aldo, 1972, Matt Krems,2007, Ibrahim,2005]:

$$\frac{\partial f^{\circ}}{\partial t} + \frac{1}{\nu} V \cdot \frac{\partial f'}{\partial t} + \frac{\nu}{3} \nabla_r \cdot f' + V \cdot \nabla_r f^{\circ} -$$

$$\frac{e}{m} \frac{1}{3\nu^2} \frac{\partial}{\partial \nu}\left(\nu^2 E \cdot f' - \frac{e}{m}\frac{1}{\nu} V \cdot \left(E \frac{\partial f^{\circ}}{\partial \nu} + B \times f'\right)\right)$$

where, ν refers the velocity, V is the volume, f° and f' are

$$= \overline{B^{\circ}} + \frac{1}{\nu} V \cdot \overline{B'} \qquad (45)$$

components of distribution functions, *e* and *m* are electronics charge and mass respectively, *E* is the electric field and *B* is the magnetic field. By using the Finite-Difference method to solve numerically the Eq.(45) to calculate the transport coefficient, namely, electron drift velocity, V_d the ratio of the diffusion coefficient to the electron mobility, D/μ and momentum transfer collision frequency, ν_m [Allen,2005]. This coefficients had been used to calculate K_1, K_T, H, θ and $\tan\theta$, then, the later quantities could be used to calculate the parameters, W, L, G and η.

3. Results and Discussion

Figs. (2-4) were showing Townsend's energy factor as a function of the E/P, where as the values of K_1, K_T are increasing with E/P because the accurate energy from the electric field. The fig.(4) refers the experimental values [Huxley,1949], when the values of K_1, obtained from measurements over a range of electric forces Z and gas pressure P are plotted versus E/P. The lower curve (Druyvesteyn's velocity distribution) derived from the upper curve (Maxwell's velocity distribution) by dividing its ordinates by 1.14. It therefore refers the values of K_T on the assumption of a Druyvesteyn distribution. The figs.(2-3) are referring the theoretical values which appeared a good agreement with the experimental values.

Figs.(5-7) are shown the drift velocity, W, as a function of the E/P. The curves are obtained by using the same set of experimental values of $\tan\theta$ in the two formula (15) and (16), one corresponding to the distribution law of Maxwell and the other to that of Druyvesteyn, it is therefore reasonable to suppose that the actual law of distribution of the velocities of agitation U is not very different from of Druyvesteyn. From the above, the theoretical values, figs.(5-6), are appearing a good agreement with the experimental values, fig.(7) [Huxley,1949].

Figs.(8-10) are showing the mean free path at unit pressure versus the mean velocity of agitation, $\overline{U}$, for velocities distribution law for Maxwell & Druyvesteyn in nitrogen gas and air, whereas expressed L in terms of experimental quantities. The values of L derived from the eqs.(43-44), using the measured values of $\frac{E\tan\theta}{H(E/P)}$ are shown in fig.(10) as functions of $\overline{U}$. It is evident that L is a function of $\overline{U}$, and therefore of the energy of agitation. The figs.(8-9) appeared a good agreement with experimental values, fig.(10) [Huxley,1949].

Figs.(11-13) are referring the mean proportion of energy η lost by an electron in collision with gas molecules as a function of K_T, in aggregate, more energy being lost in some collision than is gained in others. Since η is zero when $K_T = 1$, it following from the results that η increases with K_T to a constant value $\eta = 1.3\times10^3$ and then rises continuously as K_T is further increased. In this discussing the theory of the interaction of ratios waves in the ionosphere, that η is given by the formula (38) which makes η vanish, as it should when E=0 when the electrons are in thermal equilibrium with the gas $K_T = 1$.

The figs.(11-12) are represent the theoretical values, which appeared a good agreement with experimental values, fig(13) for Ref.[Huxley,1949].

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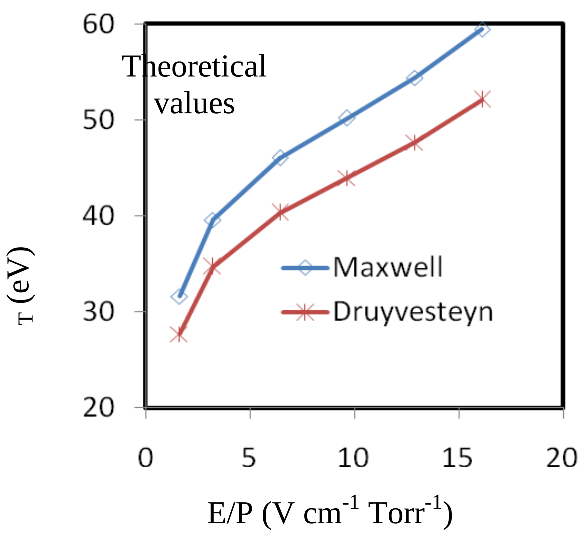

Fig.(2): The Townsend's energy factor as a function of the applied electric field to the gas pressure ratio, E/P for law of velocities distribution for Maxwell & Druyvesteyn in nitrogen gas.

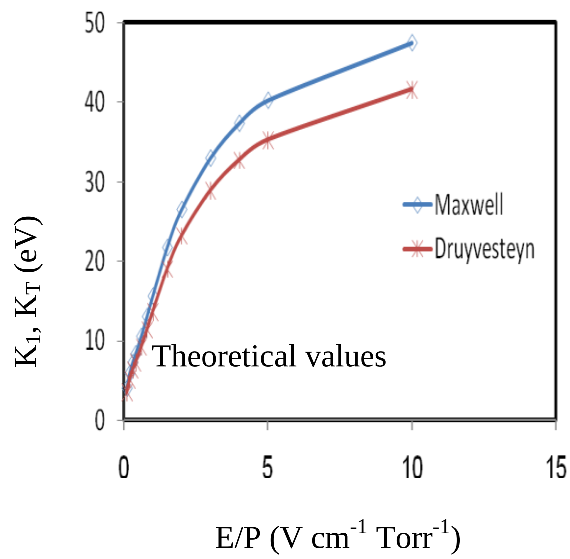

Fig.(3): The Townsend's energy factor as a function of the applied electric field to the gas pressure ratio, E/P for law of velocities distribution for Maxwell & Druyvesteyn in air.

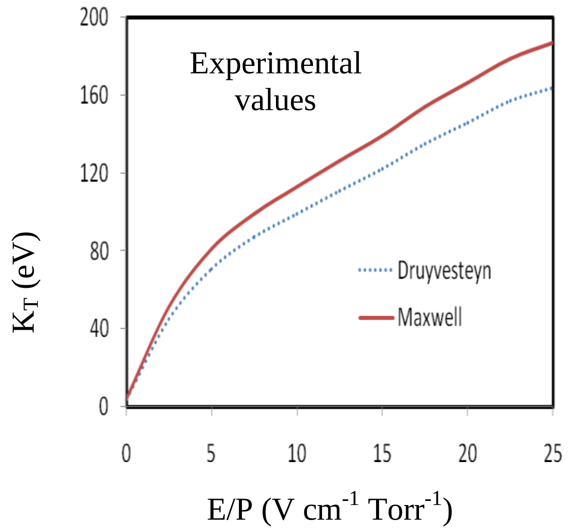

Fig.(4): The Townsend's energy factor as a function of the applied electric field to the gas pressure ratio, E/P for law of velocities distribution for Maxwell & Druyvesteyn in air.

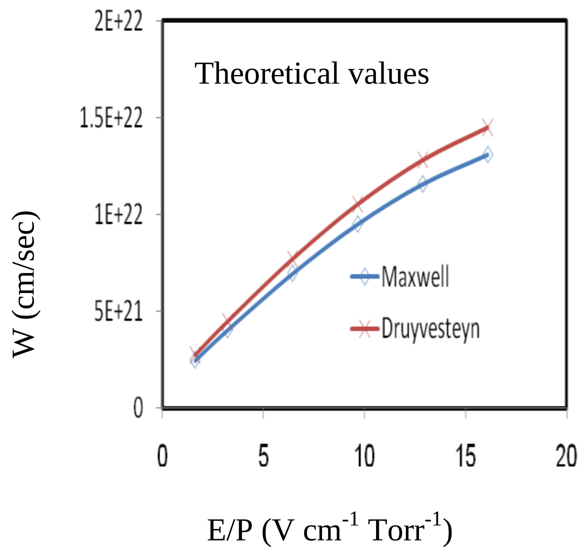

Fig.(5): The drift velocity as a function of the applied electric field to the gas pressure ratio, E/P for law of velocities distribution for Maxwell & Druyvesteyn in nitrogen gas.

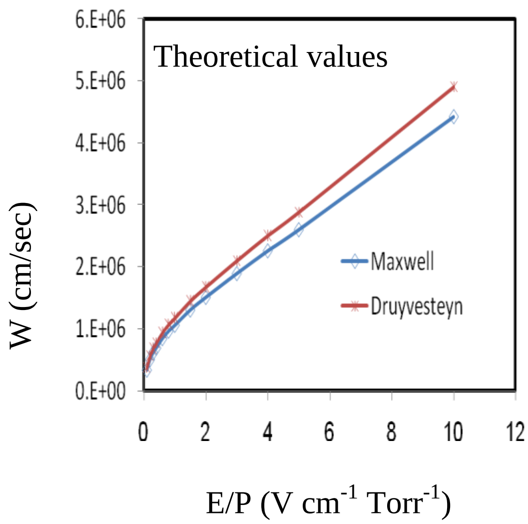

Fig.(6): The drift velocity as a function of the applied electric field to the gas pressure ratio, E/P for law of velocities distribution for Maxwell & Druyvesteyn in air.

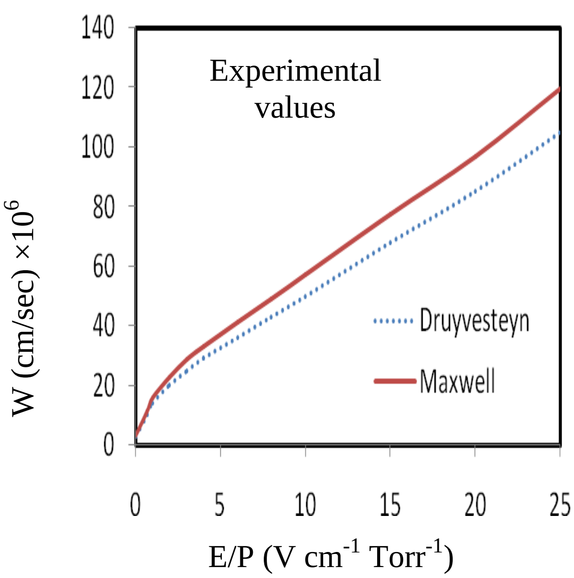

Fig.(7): The drift velocity as a function of the applied electric field to the gas pressure ratio, E/P for law of velocities distribution for Maxwell & Druyvesteyn in air.

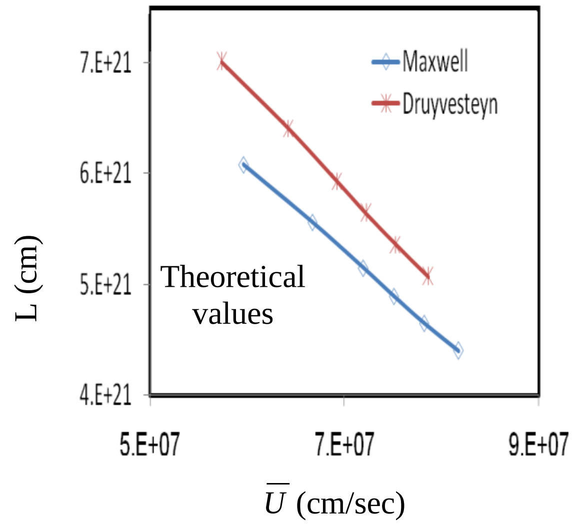

Fig.(8): The mean free path at unit pressure as a function of the mean velocity of agitation, $\overline{U}$, for law of velocities distribution for Maxwell & Druyvesteyn in nitrogen gas.

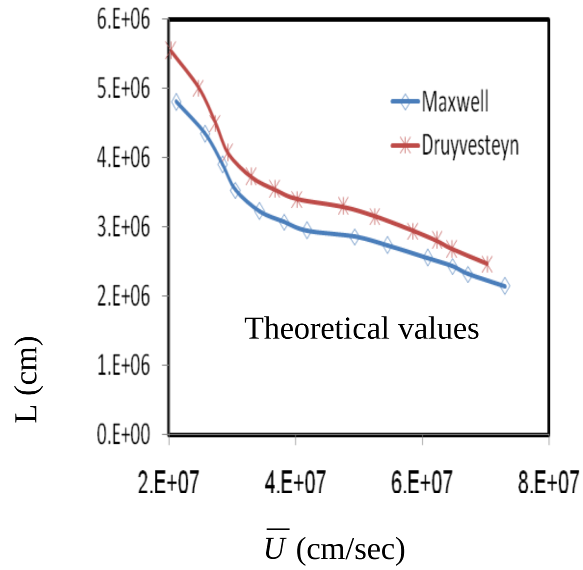

Fig.(9): The mean free path at unit pressure as a function of the mean velocity of agitation, $\overline{U}$, for law of velocities distribution for Maxwell & Druyvesteyn in air.

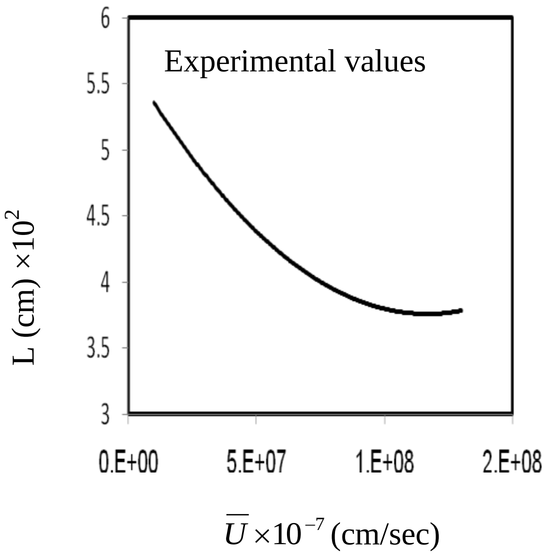

Fig.(10): The mean free path at unit pressure as a function of the mean velocity of agitation, $\overline{U}$, for law of velocity distribution for Druyvesteyn in air.

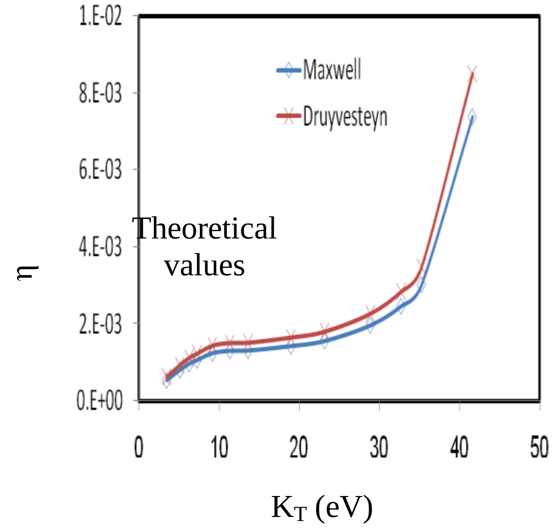

Fig.(12): The mean proportion of energy η lost by an electron in collision with gas molecules as a function of K_T, for law of velocities distribution for Maxwell & Druyvesteyn in air.

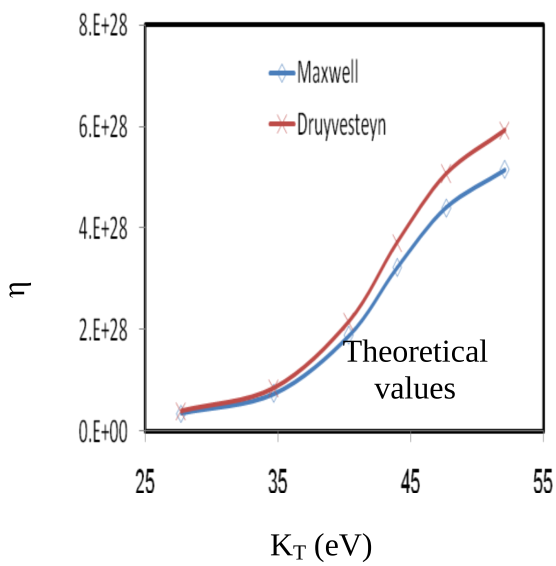

Fig.(11): The mean proportion of energy η lost by an electron in collision with gas molecules as a function of K_T, for law of velocities distribution for Maxwell & Druyvesteyn in nitrogen gas.

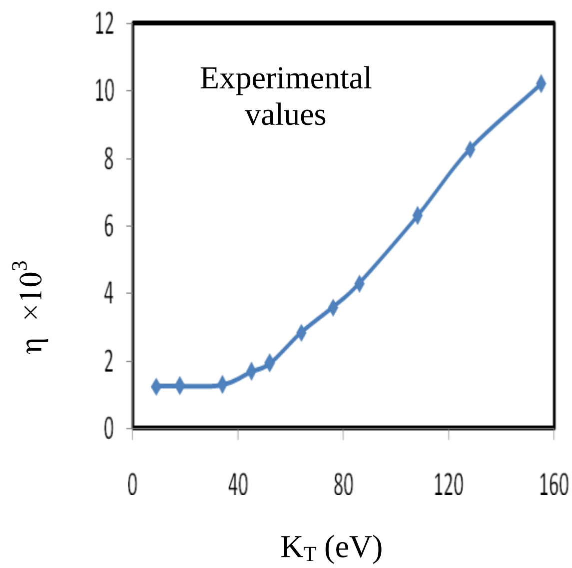

Fig.(13): The mean proportion of energy η lost by an electron in collision with gas molecules as a function of K_T, for law of velocity distribution for Druyvesteyn in air.