Transient Heat Transfer Analysis in Air Cooling of Individual Spherical Food Products

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Abstract

The one-dimensional transient heat conduction, in spherical coordinates, is solved with convective surface boundary conditions during air-cooling using separation of variable analytical method. A calculation began with transform the spherical coordinate to the rectangular coordinate to simplify the problem. The calculated temperature for apples and potatoes are compared with measured values available in the literature, and a good agreement is obtained.

Keyword: Transient heat transfer, air cooling, the freezing of foods, food products, refrigeration.

Introduction

The freezing of food is the most significant application of refrigeration. In order for freezing operations to be cost-effective, it is necessary to optimally design the refrigeration equipment. This requires estimation of the freezing times of foods and the corresponding refrigeration loads. These estimates, in turn, depend upon the surface heat transfer coefficient for the freezing operation [Brian A. Fricke, 2002].

The freezing of food products by individually quick freezing presents some difficulties due to the traderness and high perishability of these fruits. Therefore, there is a trend to pre cool them in order to increase their firmness, to produce a high quality product. Precooling improves efficiency of the freezing operation and and freezer capacity is increased due to shorter dwell times [Guemes et al, 1988].

For designing a proper per cooling system for particular food stuff a prior knowledge is gathered either by performing experiments on the product or by mathematical simulation. When fruits and vegetables are precooled, heat removed by convective heat transfer from the product surface to the cooling medium while product desiccation produces an additional cooling effect [ansari et al, 1984].

Heat transfer characteristic during air-cooling are important for a proper design and operation of such systems. Empirical equations to predict surface heat transfers are only valid when heat transfer by convection.

Hayash et al.(1975) studied the self-freezing of a wet substance were treated as a heat conduction problem were derived from the analytical procedure of Neumann solution. Cleland and earle (1977) investigated the one - dimensional heat transfer in infinite slabs of freezing food material cooled from the sides.

Ansari et al. (1984) solved one dimensional heat conduction equation in spherical-coordinate by numerical technique with convective boundary condition from the surface.Zuritz and Singh (1985) predicted temperature distribution and time-temperature history for freezing food products when subjected to fluctuation storage condition using a finite element computer model. The effects of products properties on temperature prediction were performed. Guemes et al. (1988) studied a heat transfer characteristic during air-precooling of strawberries. The effective surface heat transfer coefficients were determined and Nu-Re correlation, which includes the effect of moisture evaporation, was developed.

Ravi Kumar et al. made an experimental investigation for studying the heat transfer during forced air precooling of orange and tomato at different cooling velocities of cold air at 4–5 C^o. The air velocity has been varied from 1.2 to 4.4ms^{-1}. It was found that air velocity had significant bearing on the cooling rates of food products below the dimensionless temperature, T, of 0.6. The limiting cooling air velocity for orange was 3.5ms^{-1} and that for tomato was 2.6ms^{-1}[Ravi Kumar et al., 2007].

In the present work a new analytical calculation is proposed using a separation of variable technique to calculate the time-temperature behavior during air-cooling food products.

Analytical approach

The one-dimensional heat conduction equation in the spherical coordinate (r ,t) only given as:

$$\frac{\partial^2 U}{\partial r^2} + \frac{2}{r}\frac{\partial U}{\partial r} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad (1)$$

Or

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(rT) = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad (2)$$

Ozisik (1976) given a procedure to convert the spherical coordinate to the rectangular coordinate by using a new variable defined as:

$$T(r\,,t) = rU(r\,,t) \qquad (3)$$

Then, Equation (2) is transformed into:

$$\frac{\partial T}{\partial t} = \alpha\frac{\partial^2 T}{\partial r2} \qquad (4)$$

The problem solved by using separation of variable technique as follows:

Let:

$$T(r\,,t) = f(t)g(r) \qquad (5)$$

Combination of equation (4) and equation (5) gives:

$$f'(t)g(r) = \alpha f(t)g^{"}(r) \qquad (6)$$

Or

$$\frac{f'(t)}{\alpha f(t)} = \frac{g''(r)}{g(r)} = \pm\lambda \qquad (7)$$

The problem under study is satisfying a negative sign of λ. Then equation (7) becomes:

$$\frac{f'(t)}{\alpha f(t)} = \frac{g''(r)}{g(r)} = -\lambda \qquad (8)$$

Now,

$$f'(t) = -\lambda\,\alpha f(t) \qquad (9)$$

And

$$g''(r) = -\lambda g(r) \qquad (10)$$

Equation (9) is satisfied by the exponential function as:

$$f(t) = c\exp(-\alpha\lambda^2 t) \qquad (11)$$

And Eq(10) is satisfied by a sine and cosine functions as:

$$g(r) = A\,cos(\lambda r) + B\sin(\lambda r) \qquad (12)$$

Substitute equation (11) and equation (12) into equation (5) yield:

$$T(\lambda\,,r) = [A\,cos(\lambda r) + B\sin(\lambda r)\,]\exp(-\alpha\lambda^2 t)\,] \qquad (13)$$

The boundary condition is:

at r = 0 $\frac{\partial T}{\partial r} = 0$ (14)

And at r = R $\frac{\partial T}{\partial r} = -BiT$ (15)

Differentiation equation (13) and introduction of the boundary condition from equation (14) result B=0 [Carslaw and Jaeger, 1959] , hence equation (13) becomes:

$$T(r\,,t) = A\,cos(\lambda r)\exp(-\alpha\lambda^2 t) \qquad (16)$$

The boundary condition from equation (15) [at r=R] can be written as:

$$\frac{\partial T(r\,,t)}{\partial r} + BiT = 0 \qquad (17)$$

After introduction equation (16) we get:

$$\Psi\tan\Psi = BiR \qquad (18)$$

Where

R (19)ψ=λ

The initial condition will be fulfilled if the solution can be developed in finite series [Carslaw and Jaeger, 1959] as:

$$\frac{T(r,t)}{T_0} = \sum_{n=1}^{\infty} A_n \cos\left(\frac{\psi_n\ r}{R}\right) exp(-\alpha\psi_n^2 t\ /R^2) \quad (20)$$

[Carslaw and Jaeger, 1959] given the value of A_n as :

$$A_n = \frac{4\sin\psi_n}{(2\psi_n + \sin(2\psi_n))} \quad (21)$$

Table IV in Carslaw and Jaeger , 1959 indicate that for BiR< 0.1 the second root ψ_2 of Eq (18) is greater than ten times the first root ψ_1 and thus, the series is rapidly converging. In this range A_1 is also very close to unity. With decreasing BiR , the coefficient A_1 tends to unity and the higher A_n coefficient tend to zero . According to equation (20) at r=0 and t=0 the temperature ratio is determined by the sum of the A_n coefficient only. Since the initial temperature ratio should be equal to unity. With growing time t, convergence improves because of the second and subsequent terms are more suppressed than the first one. [Botros et at. 1989]. Therefore the calculation of the transient temperature using the first term of equation (20) is sufficient and hence;

$$\frac{T(r,t)}{T} = \exp\left[\frac{-\alpha\Psi^2 t}{R^2}\right] \quad (22)$$

Results and Discussion

The time – temperature analytical solution for two apples and two potatoes was presented. It was found that the present solution with only heat transfer has lower values than the experimental and other models available in literature. The reason may be coming from that this model assumes no mass transfer and negligible the effect of moisture evaporation phenomena which is a complex one. It depends on a number of parameters like the cooling air temperature, its humidity, and water content of products, internal mass diffusion , skin effect and the temperature gradients [Ansai et al., 1984].

The results figs (1, 2, 3 and 4) showed that at the beginning of the cooling of the food products the present model predicted surface temperature are slightly different from experimental values. The reason may be due to the during this period the product lose their surface moisture, and the evaporation cooling effect become very important. Another reason for this difference is that the true surface area of apples and potatoes might by higher than the sphere which is assumed in the analysis. The last reason of the difference may by mainly attribute to the additional cooling due to water evaporation from the food products.

All figures indicated a good agreement between the presented model results and the available results given by Ansai et al. (1984).

Conclusions

An analytical solution of heat transfer from an apples and potatoes like a sphere as we assumed given the following conclusions:

1. The assumption of uniform sphere shape of apples and potatoes gives good results.

2. The effect of moisture appearing in the earlier stage of cooling process and then dispersed.

3. The symmetrical boundary conditions are sufficient to predict the temperature time behavior of the cooling food products.

4. A very good agreement between the presented model results and available results given by Ansari et al. (1984).

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Nomenclature

A integration constant

B integration constant

Bi Biot Number (dimensionless)

α thermal diffusivity (m^2/sec)

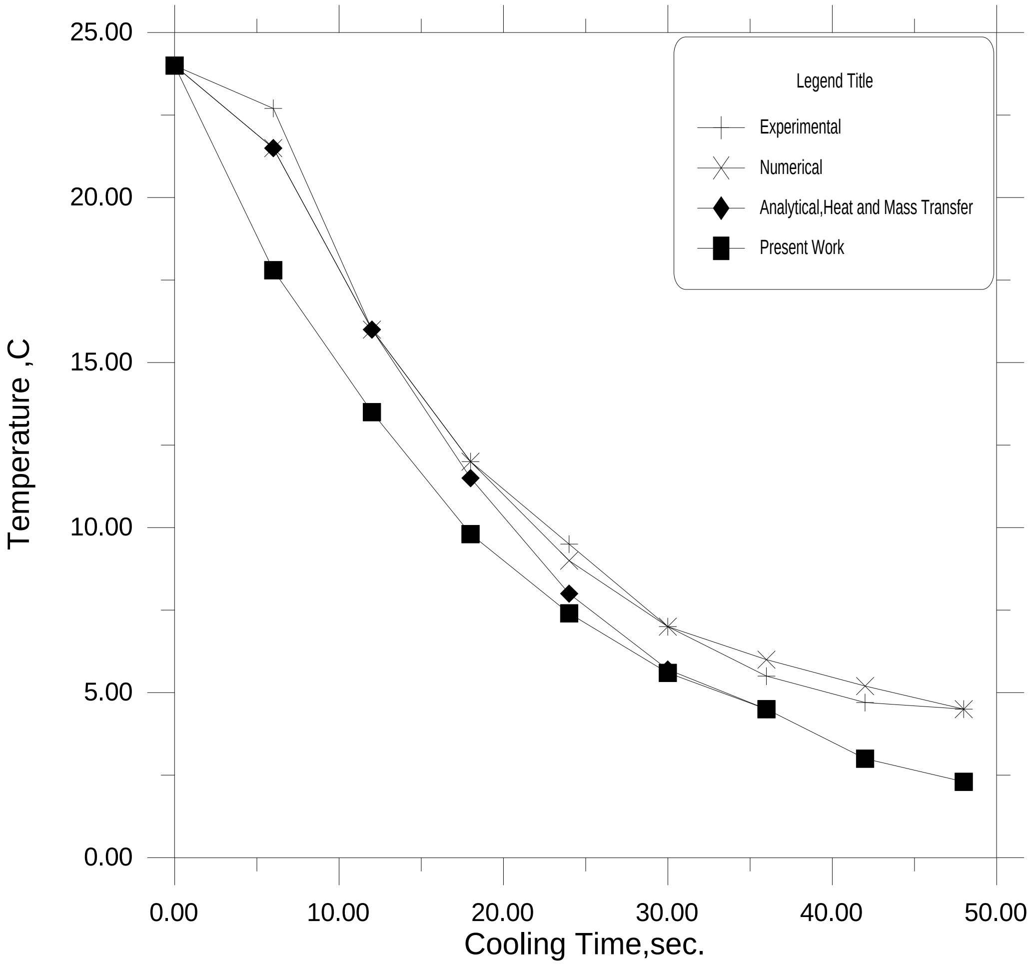

Fig.(1) Time-Temperature Plots of Apple-1, R=0.0388 m, r=0.01358 m, density=803 kg/m3

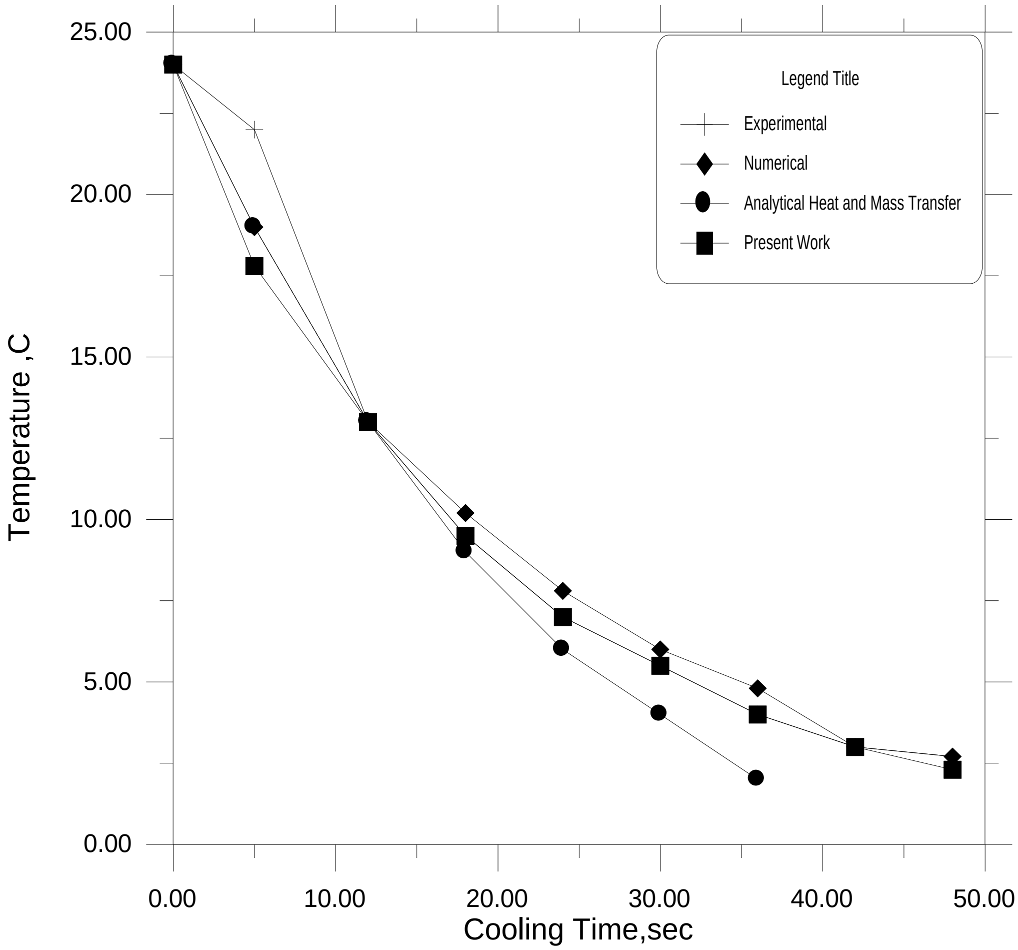

Fig.(2) Time-Temperature Plots of Apple-2, R=0.0354 m, r=0.014 m and density=807 kg/m3

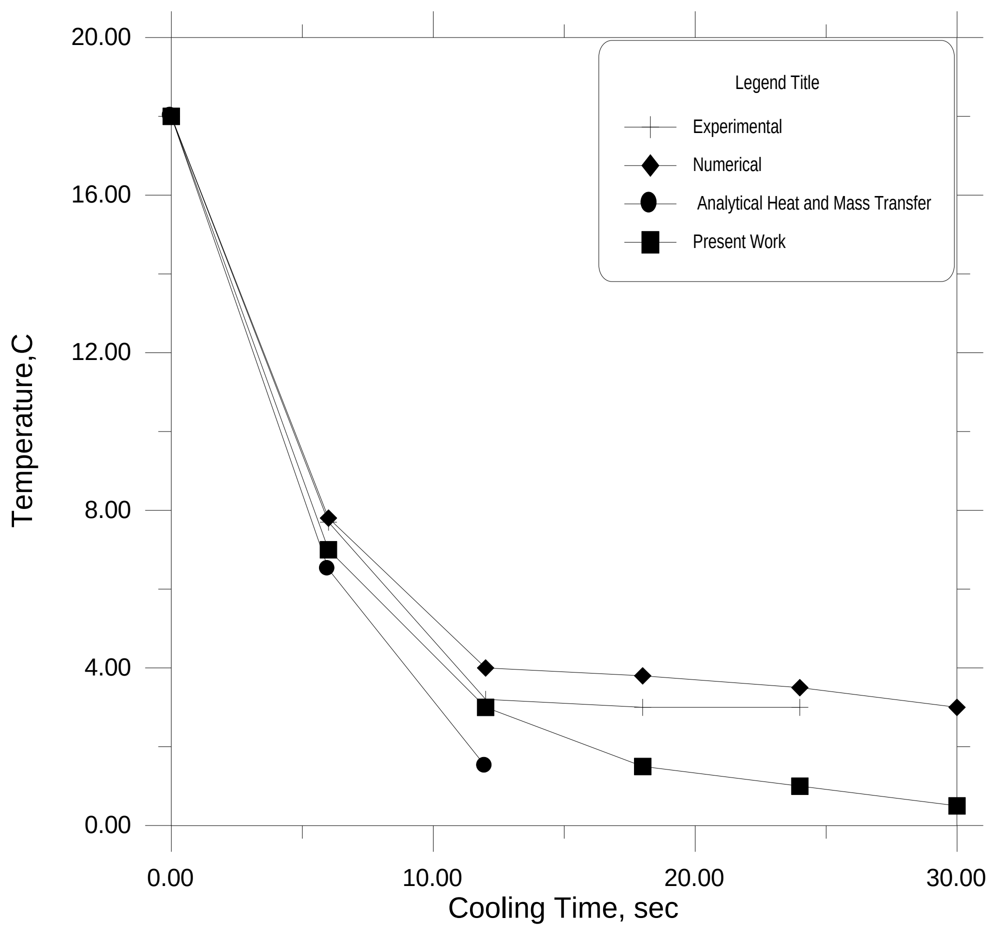

Fig.(3) Time-Temperature Plots of Potato-1, R=0.022 m,r=0.00561 m and Density=1150 kg/m3

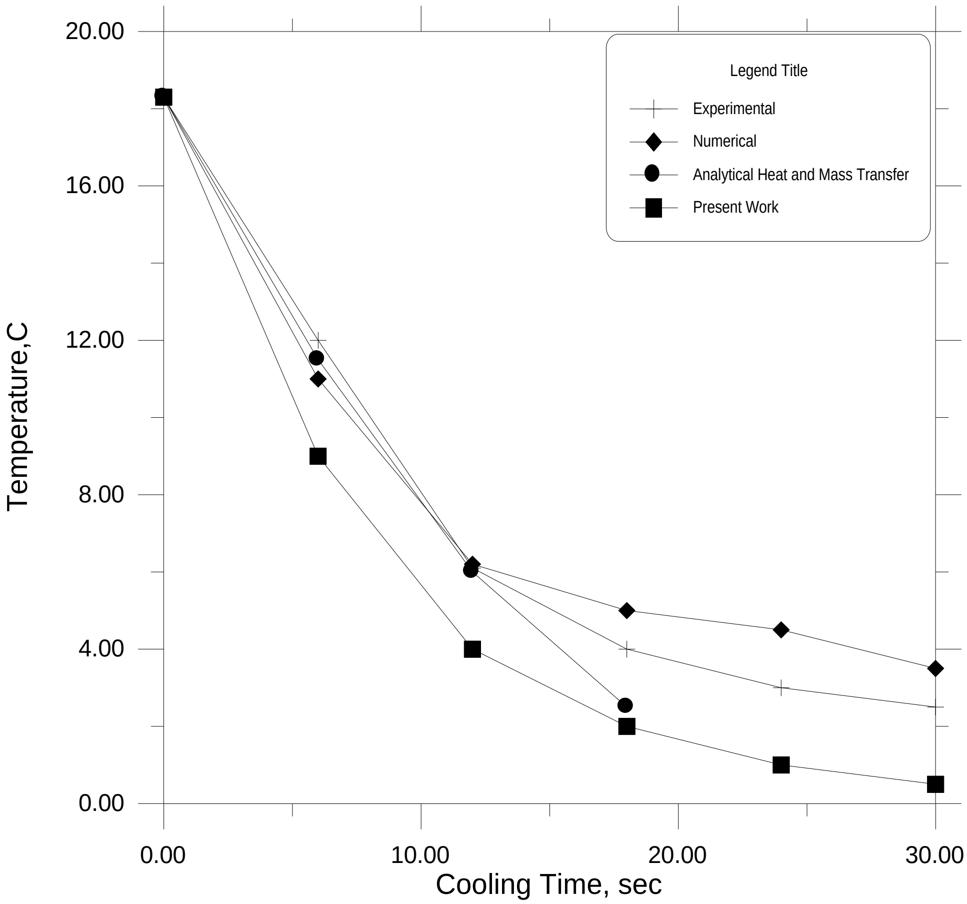

Fig.(4) Time-Temperature Plots of Potato-2 , R=0.0285 m , r=0.0701 m and Density=1135 kg/m3