Theoretical Analysis of Eccentric Flow of Cylindrical Bodies in Pipelines

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Abstract

Predictions of pressure gradient ratio for the flow of cylindrical capsule moves eccentrically respect to the pipe. Results were obtained for the transport by water of single cylindrical capsules with density ratio of 1.2, 1.5, and 2.0 in a 0.046 m pipe. The diameter ratios varied from 0.5 to 0.9 .The average flow velocity varied from 0.3 to 0.5m/s. The increase in average flow velocity led to decrease the pressure gradient ratio, the increase in diameter ratio increases the pressure gradient ratio and the increase in density ratio increases the pressure gradient ratio . A good agreement between the theoretical results obtained by the present work with that obtained by empirical relationship [Liu, 1981].

Keywords: capsule flow, capsule pipeline, freight pipeline, solid liquid flow, hydraulic pipeline.

1. Introduction

Capsule pipeline is the transport of freight (solids) in capsules (containers or vehicles) moving through pipelines. When the fluid used for propelling the capsules in the pipe is in or another gas, it is called pneumatic capsule pipeline (PCP); when the fluid for propelling the capsules is water or another liquid, it is called hydraulic capsule pipeline(HCP).Both PCP and HCP have distinct characteristics and have niches or "windows of opportunity "[Liu,2001]. The word “capsule" generally means a container and its contents ; however , for convenience the word "capsules" is used also as a general term to include all shaped solids transportable in pipelines , whether hollow or solid, cylindrical or spherical and of any density.

Capsules with a density less than water would be expected to behave in a similar way to capsules denser than the water for a given density differential [Ellis, 1964]. The forces acting on the capsules when a horizontal pipeline is [Vijay, 1987]:

1-The pressure forces in the fluid acting at every point normal to the capsule surface (these forces vary both in the radial and axial directions);

2- The shear forces due to the fluid acting at every point parallel to the capsule surface;

3- The weight of the capsule acting vertically downwards;

4- A friction force caused any contact between the capsule and the pipeline.

Tomita [Tomita, 1986] analyzed numerically the motion of a single capsule in a hydraulic pipeline by using the method of characteristics .Liu [Liu, 1987] studied analytically and experimentally the behavior of non-uniform-density capsules in hydraulic capsule pipeline. Also Liu [Liu, 1982] give a theory to explain the lift on cylindrical capsules in full suspension in water flowing through pipes. The theory differs from the contemporary explanation using a thrust bearing analogy which holds only when laminar flow exists in capsule/pipe clearance. Charles [Charles, 1962] derived a theoretical model to predict the velocity and pressure gradient for the flow of long cylindrical capsule moves concentrically with respect to the pipe.

An analytical solution and a numerical analysis are presented to study the flow Behavior in concentric annulus with moving core in pipe for laminar flow condition. The analytical analysis is presented as exact solution for steady, fully developed and one dimensional flow. The numerical model is presented to study two-dimensional, steady, developing and fully developed flows. The numerical model established a staggered grid for axial and radial velocities (Vz and Vr) and using the pressure correction technique [Mohamed, 2008].

The velocity ratio of heavy density spherical, cylindrical, square and rectangular capsules is greatly dependent on the ratio of characteristic diameter to the diameter of capsule and the shape factor. The velocity ratio of heavy density cylindrical capsules having noses of different shapes is found to be dominantly dependent on the ratio of location of mass center of the capsule to the diameter of capsule[Agarwal,2001].

In the present paper, we will differentiate a theoretical model of the pressure gradient caused by the flow of single cylindrical capsules through a horizontal pipeline with density greater than that of the water.

2. Theoretical Analysis

Referring to figure (A), we may write force balances over a length of pipe, Lc, containing a single capsule, as follows:

a. for the total contents of the pipe,

$$\frac{\pi D^2 \Delta P_c}{4} = L_c \pi D \tau_{wa} + \eta_m W_B \qquad (1)$$

b. for the capsule itself,

$$\frac{\pi d^2 \Delta p_c}{4} = L_c \pi d \tau_{ca} + \eta_m W_B \qquad (2)$$

Where

$$W_B = \frac{\pi d^2 (\rho_c - \rho) L_c}{4} g \qquad (3)$$

Equation (1) may be written as

$$\left(\frac{\Delta P}{L}\right)_c = \frac{4\tau_{wa}}{D} + \frac{4}{\pi D^2 L_c}\eta_m W_B \quad (4)$$

Or, introducing the relation of equation (3), as

$$\left(\frac{\Delta P}{L}\right)_c = \frac{4\tau_{wa}}{D} + \eta_m k^2 g(\rho_c - \rho) \quad (5)$$

Equation (2) becomes:

$$\left(\frac{\Delta P}{L}\right)_c = \frac{4\tau_{ca}}{d} + \eta_m g(\rho_c - \rho) \quad (6)$$

This is an eccentric case, because the capsule density is different from the fluid density.

To a first approximation ,we could assume that the average wall shear stress, τ_{wa} ,would be the same as in concentric capsule case at the same average flow velocity, u_{av} ,but based upon the velocity profile assumed by Charles [Charles,1962].With this assumption the term $4\tau_{wa}/D$ becomes identical with $(\Delta P/L)_C$ for concentric capsule case ,here designed as $(\Delta P/L)_C^*$ .It may therefore be expressed as:

$$\frac{4\tau_{wa}}{D} = \left(\frac{\Delta P}{L}\right)_C^* = R_P^* \left(\frac{\Delta P}{L}\right)_L \quad (7)$$

Thus R_P^* can be calculated by using one of the following equations derived by Charles [7]:

$$R_P^* = \frac{1}{1 - K^4} \quad (8)$$

$$R_P^* = \left[\frac{0.82}{(1-K^2)^{1/7}\left(\frac{7}{4}K(1-K)+\frac{49}{60}(1-K)^2+K^2\right)}\right]^{1.75} \qquad (9)$$

$$R_P^* = \frac{202}{(1-K^4)\left(\frac{Du_{av}\rho}{\mu}\right)^{0.75}} \qquad (10)$$

Where $R_P^* = \frac{(\Delta P/L)_C}{(\Delta P/L)_L}$

If the free liquid stream is in turbulent flow, $(\Delta P/L)_L$,may be calculated at any value of V_M from the following equation [Blasius equation]:

$$\left(\frac{\Delta P}{L}\right)_L = 0.158\left(\frac{\mu}{Du_{av}\rho}\right)^{0.25}\frac{u_{av}^2\rho}{D} \qquad (11)$$

3. Results and Discussion

Result were obtained for the pipe diameter equal to 0.046,capsule/pipe diameter ratio varying from 0.5 to 0.9 , the average flow velocity ranging from 0.3 to 0.5 $\frac{m}{s}$, density ratio varying from 1.2 to 2.0, and $\eta_m = 0.3$

Figures (1-5) represent the relationship between the average flow velocity and pressure gradient ratio at constant density ratio, in which the increasing in average flow velocity at constant diameter ratio will decrease pressure gradient ratio because the presence of the capsule will make the flow velocity of the mixture (capsule - fluid) less than the flow velocity for the flow of fluid alone ,this due to the contact of the capsule surface with the pipe wall.

Figure (6) shows the effect of diameter ratio on the pressure gradient ratio at constant average flow velocity and density ratio, in which the increase in diameter ratio will increase the presser gradient ratio, this due to the fact that the increase in diameter ratio will make the capsule in more concentric position in the pipe i.e. it will increase the obstruction to the flow of fluid, therefore, the pressure gradient due to the capsule will decreases.

Figure(7) shows the relationship between the density ratio and the pressure gradient ratio at constant average flow velocity and diameter ratio, in which the increase in density ratio will increase the pressure gradient ratio this due to the fact that the increase in density ratio will increase the eccentricity of the flow of capsule i.e. it will be near the bottom of the pipe wall, therefore, the contact between the capsule surface and the pipe wall will increases, thus, the pressure gradient ratio will be greater.

4. Conclusions

The following conclusions were drawn from the present work:

1. The increase in average flow velocity decreases the pressure gradient ratio.

2. The increase in diameter ratio increases the pressure gradient ratio.

3. The increase in density ratio increases the pressure gradient ratio.

4. A goods agreement between the theoretical results obtained from the present work with that obtained by using the empirical relationship [Liu, 1981].

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Nomenclature

d=capsule diameter (m)

D=pipe diameter (m)

g=gravitational constant (m/s^2)

k=diameter ratio

L_c=capsule length (m)

$\left(\frac{\Delta P}{L}\right)_c$ =pressure gradient caused by capsule (N/m^3)

$\left(\frac{\Delta P}{L}\right)_L$ = pressure gradient for the flow of fluid alone (N/m^3)

R_P= pressure gradient ratio=$\frac{(\Delta P/L)_c}{(\Delta P/L)_L}$

R_ρ=density ratio=$\frac{\rho_c}{\rho}$

u_{av}=average flow velocity (m/s)

W_B =the buoyed weight of the capsule (N)

Greek Letters

η_m=the coefficient of moving friction

Dynamic viscosity of the fluid $(\frac{N}{m^2}.s)=\mu$

Fluid density$(kg/m^3)=\rho$

ρ_c=capsule density(kg/m^3)

τ_{wa}=average shear stress in the fluid at the pipe wall(N/m^2)

τ_{ca} =average shear stress in the fluid at the capsule(N/m^2)

Subscript

C=capsule

L=liquid

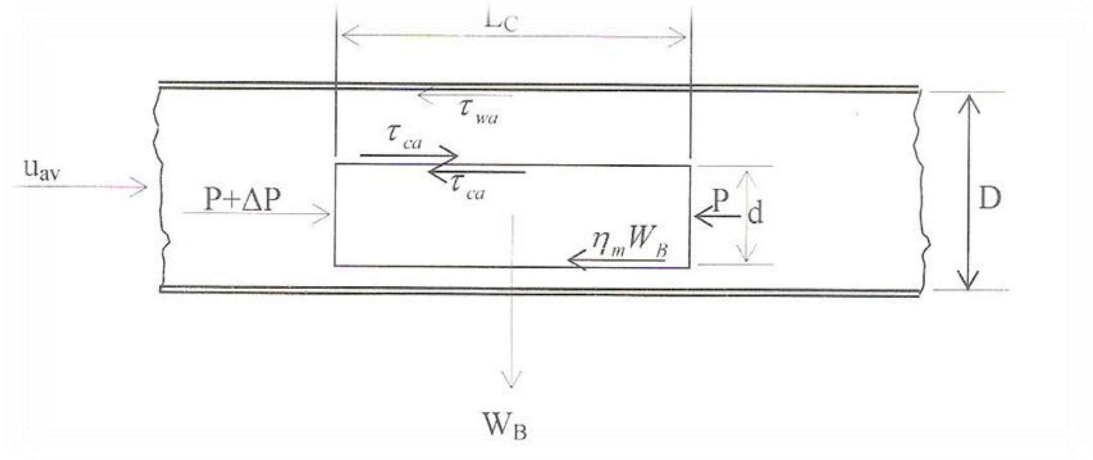

Fig.(A) Force acting on a cylindrical capsule in motion through horizontal pipeline.

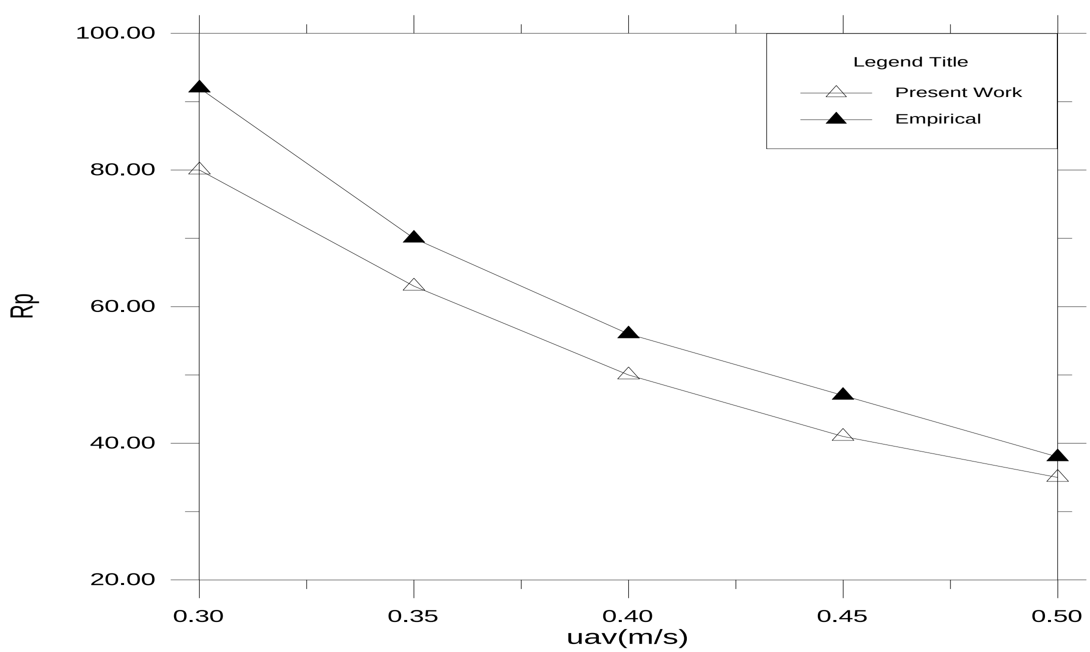

Fig.(1)Variation of pressure gradient ratio with average flow velocity for k=0.9

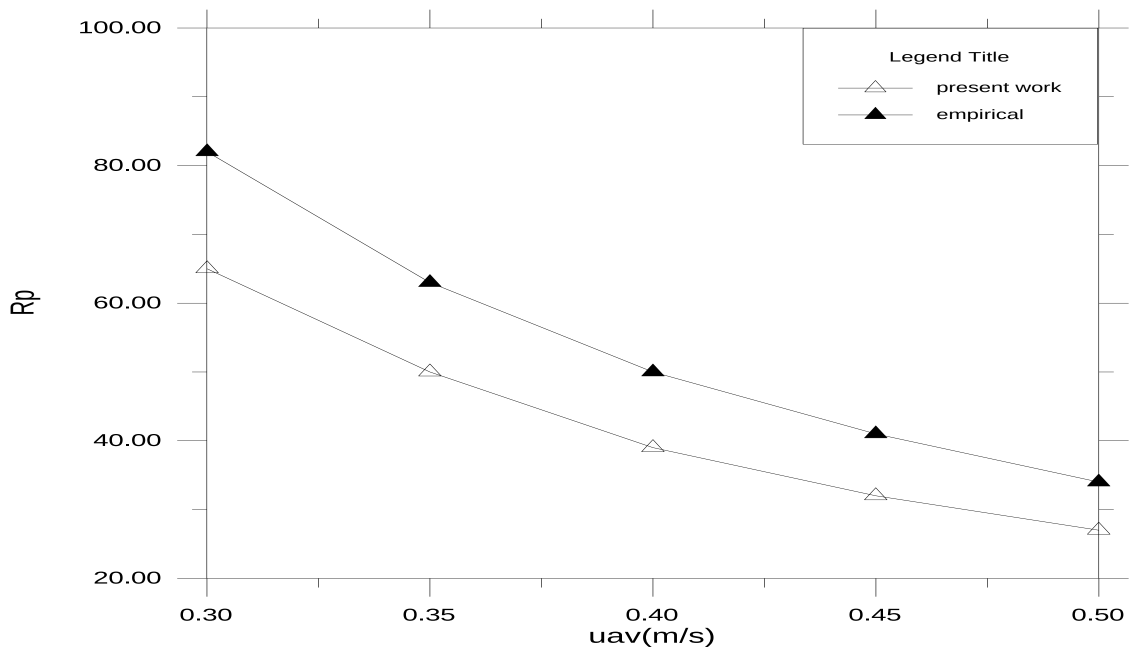

Fig.(2) Variation of pressure gradient ratio with average flow velocity for k=0.8

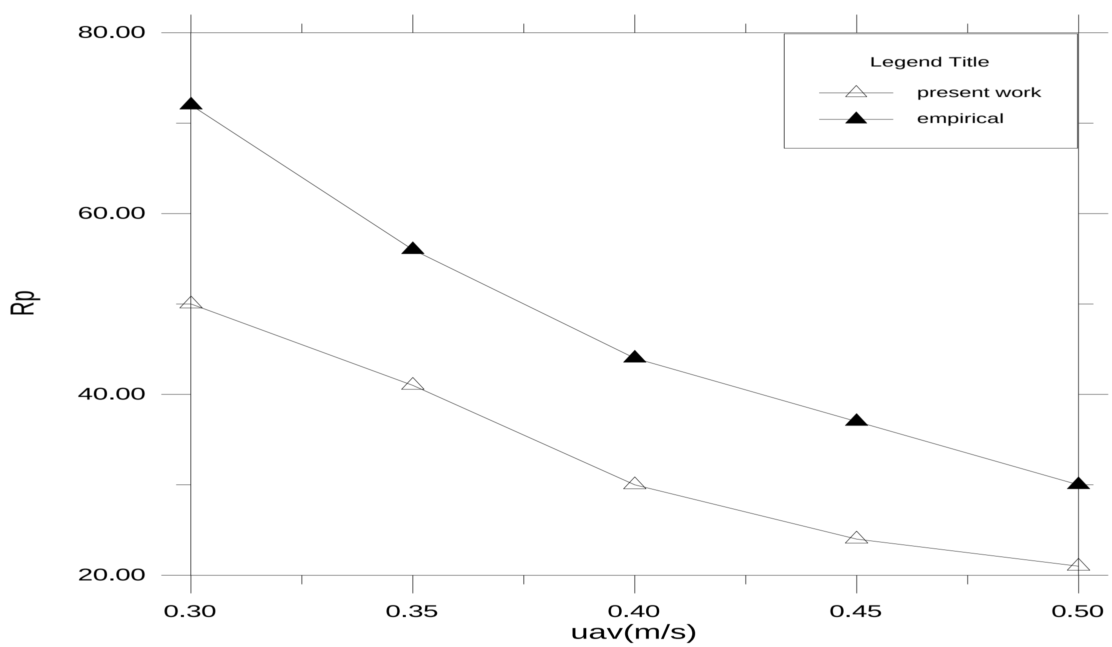

Fig.(3) Variation of pressure gradient ratio with average flow velocity for k=0.7

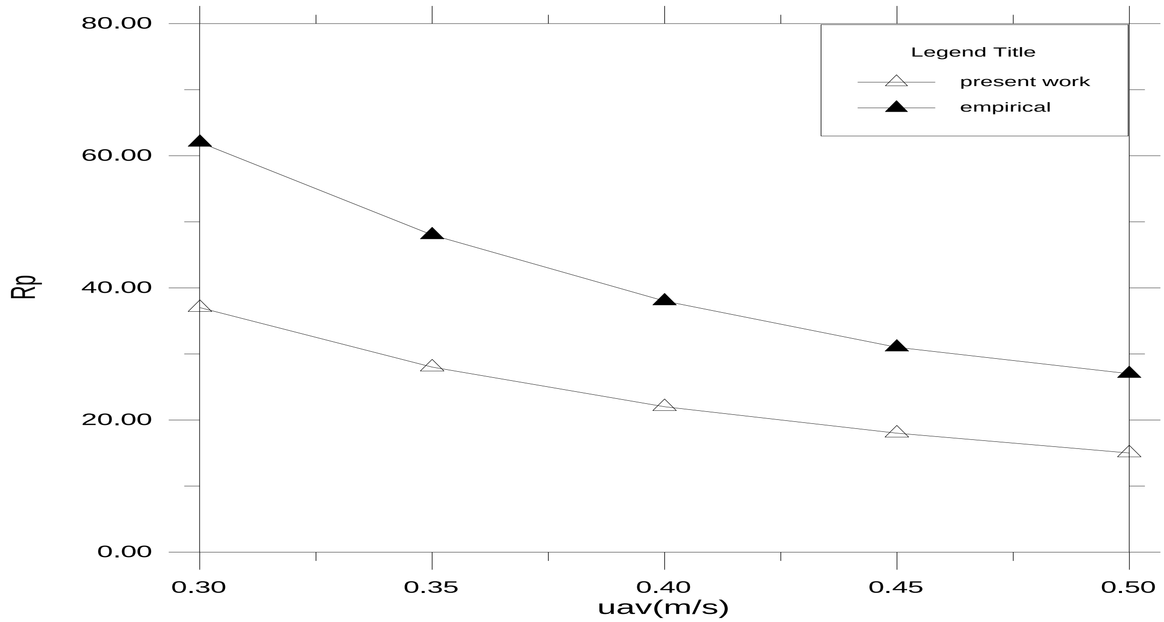

Fig.(4)Variation of pressure gradient ratio with average flow velocity for k=0.6

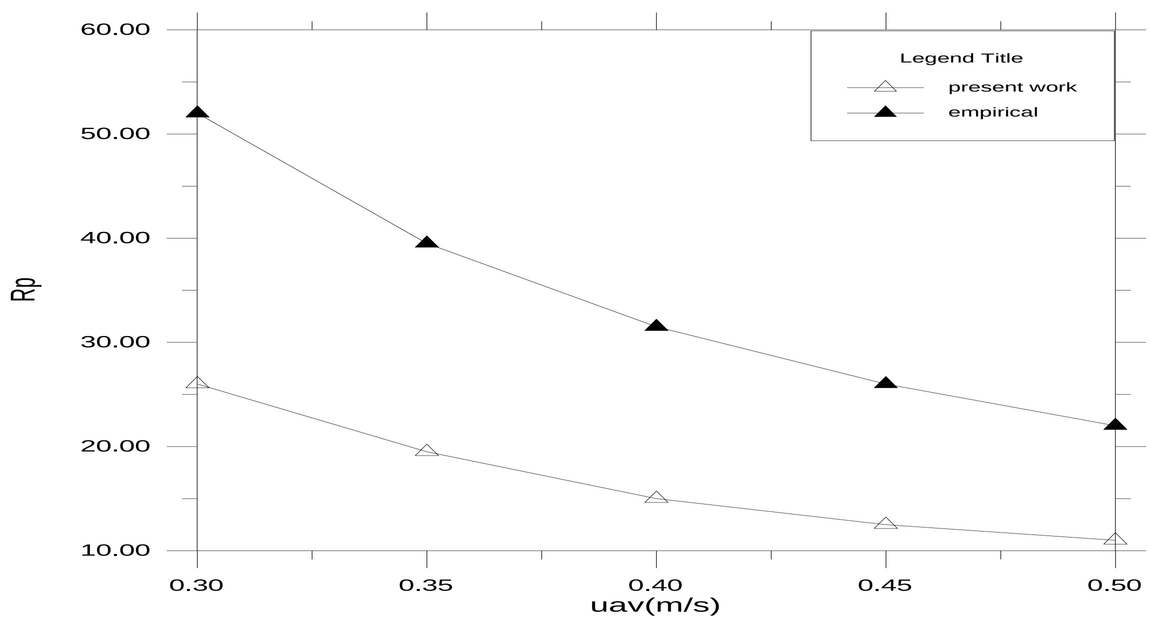

Fig.(5) Variation of pressure gradient ratio with average flow velocity for k=0.5

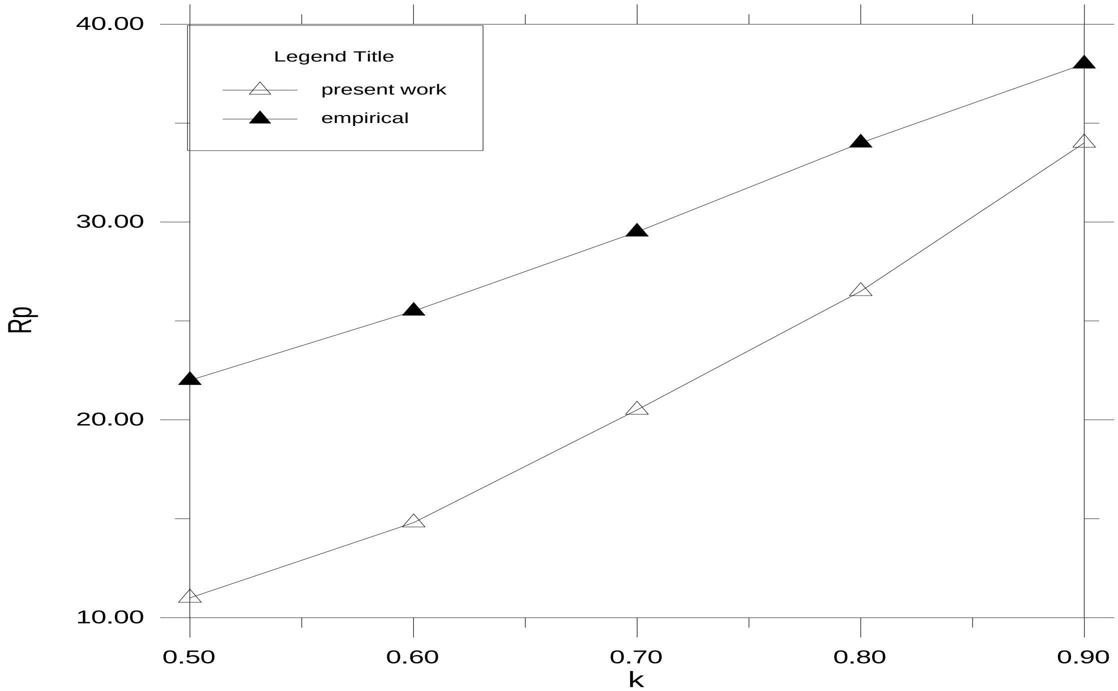

Fig.(6) Variation of pressure gradient ratio with diameter ratio at uav=0.5 m/s

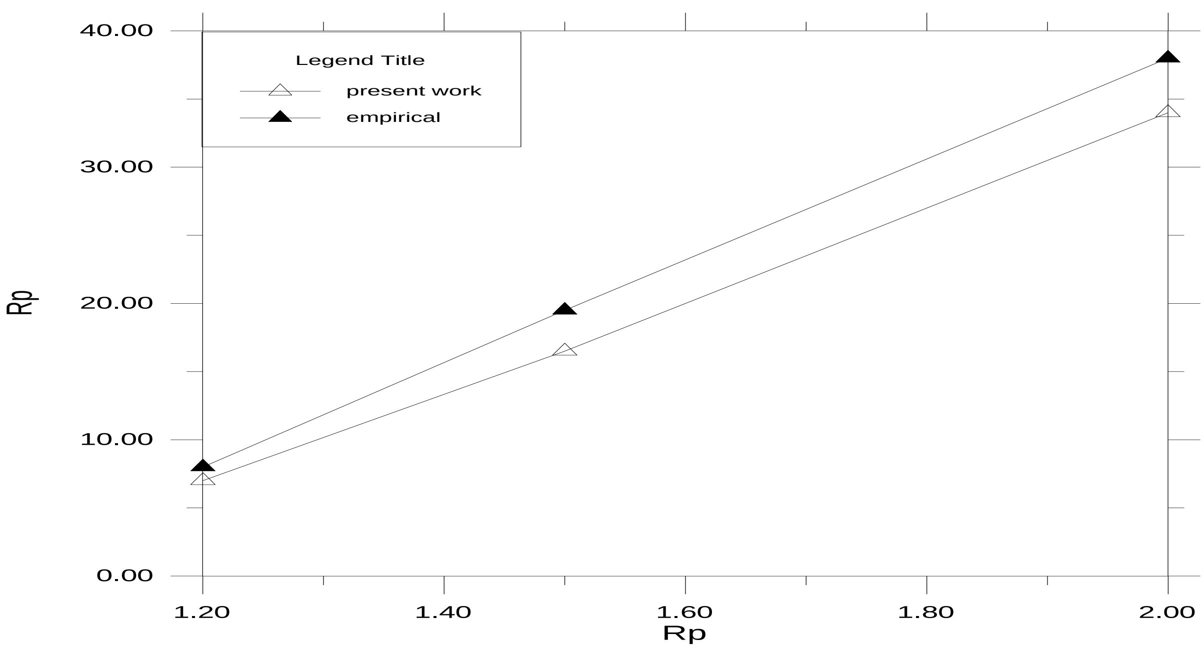

Fig.(7) Variation of pressure gradient ratio with density ratio at uav=0.5 m/s