Comparison of the Group and Intercept Coefficient from HGLM-DIF and LR-DIF Methods

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Abstract

Hierarchical linear model (HLM) studies have been widely used in social science and educational research. The pivotal aim of the study is the comparison of the similarity between group coefficients and intercept that are gained from the analyses of logistic regression and generalized gradual linear models used in adjustment of DIF. Selection of study samples was made through a stratified random sampling method, and data were gathered from 10727 students. The data used for this study consist of 25 items in the Turkish subtest items of the SSIE that were applied in Turkey in 2006. The empirical investigation compares the HGLM-DIF method with logistic regression. The DIF method has provided evidence for the assertion that the HGLM-DIF procedure is equivalent to the LR-DIF method. The similarity between these two methods has been identified in various aspects of the results, including correlation of group and intercept coefficients. The correlation of group and intercept coefficients from the two methods is quite perfect. HGLM-DIF may be strongly recommended for DIF analysis, if the outcome variable is binary, because the HGLM-DIF procedure does not require more effort and more time than the LR-DIF procedure does.

Keywords: odds-ratio coefficients; intercept and group coefficients; general hierarchical linear modeling; logistic regression

Hierarchical linear model (HLM) studies have been widely used in social science and educational research (Wang, 2000; Kim, 2003; Greer, 2004; Vaughn, 2006; Park, 2005; Acar, 2009). Also, the HLM is an extension of the basic linear regression equation (Bryk & Raudenbush, 1992). In HLM, because the statistical models include multilevels, the lowest level of analyses, level 1, and the next one, level 2, have

been offered. In studies with hierarchical data, level one represents the individual's level, and level 2 represents the group's level.

As long as it is presumed that there are different lines of regression in each group, the number of observations in more than one different group parameters and hybrid parameters that have plenty of features can be easily modeled in HLM (Gokiert & Ricker, 2004).

If the outcome variable is measuring results in ordering or binary, hierarchical general linear model (HGLM), which is a special form of HLM, can be used when the outcomes of binary variables are either true or false, and the usual linear model that assumes a normal random error fails (Kim, 2003). In the outcome variables having two categories, a binomial distribution is taken into account, which is known as a Bernoulli distribution, and the logit connection function is used (Raudenbush & Bryk, 1986). The logit connection function, which is used for the binary outcome variable, is used in this way:

$$\eta_{ij} = \log\left(\frac{\varphi_{ij}}{1-\varphi_{ij}}\right)$$

φ_{ij} in the equation is showing the probability of "to be" of the outcome variable and the outcome variable takes the values between 0 and 1. η_{ij} , is the logarithm of the probability of "to be" (log-odds).

Predictive variables are added to model level 2 that reflect the specifications of the student—this is the DIF determining performance on the item—when it is needed to examine whether the student specifications have impacts on answering the test items correctly or not. In HGLM, level 1 and level 2 equations that will be established to determine DIF with conditional modeling is presented below (Williams, 2003):

Level 1 equation (item level): to show the i (i = 1, 2,….k) item and j (j = 1,2,….N) individual index

$$\eta_{ij} = \log\left(\frac{P_{ij}}{1-P_{ij}}\right) = \beta_{0j} + \beta_{1j}X_{1ij} + \beta_{2j}X_{2ij} + \ldots + \beta_{(k-1)j}X_{(k-1)ij} + r_{ij}$$

η_{ij} : Estimated outcome variable, the probability of individual j to give the correct answer to item i.

$X_{q_{ij}}$: is an indicator variable for item i. When the answer given to an item is on item i. ($q = i$), the value is 1; in another condition ($q \neq i$), the value is 0.

β_{0j} is the intercept. When the all $X_{q_{ij}}$'s become 0, the effect of the item, which is not taken for the model, occurs. For this reason, β_{0j} is the effect of the item, which is not taken for the model.

β_{1j} is the effect of the i item on the probability (outcome variable) of individual j to give the correct answer up to i = 1,2,…(k-1). Parameters from β_{1j} to $\beta_{(k-1)j}$ are coefficients that show the effects of the items on the probabilities of giving the correct answer for the individual from 1 item to item k. Individual j is associated with different individuals and different item level parameters. If the level increases, j in β_{ij} decreases, and the item parameters are kept constant between individuals.

Level 2 is formed to see the differences between the probabilities of answering each item correctly according to the group variable of the students (Williams, 2003).

Level 2 (student level) equation:

$$\beta_{0j} = \gamma_{00} + \gamma_{01}(Group\ Variable)_j + u_{0j}$$

$$\beta_{1j} = \gamma_{10} + \gamma_{11}(Group\ Variable)_j$$

$$\beta_{(k-1)j} = \gamma_{(k-1)0} + \gamma_{(k-1)1}(Group\ Variable)_j$$

β_{ij} : is the effect of item i on the probability of giving the correct answer for individual j up to I = 1,2,...(k-1). Parameters from β_{1j} to $\beta_{(k-1)j}$ are the effects of the items on the probability of giving the correct answer form the 1 item to item k for individual j.

γ_{00} : is the referred item parameter.

γ_{01} : is the effect of the probability of correctly answering item i on group variable.

u_{0j} **:** is the effect of random group variable. It is the random effect of β_{0j}, which shows normal distribution that has distribution average 0 and variance τ .

The hierarchical generalized linear model for DIF analysis (HGLM-DIF) extends from the two-level hierarchical item analysis model by Kamata (2001). The HGLM-DIF studies will be shown to be equivalent to logistic regression DIF method (Kamata, 2001; Kim, 2003; Acar, 2008). The logistic model uses the logit of the outcome variable and group variable. Item score and the matching variable are the basic components in DIF analysis. If the performances of the group members on an item are estimated with logistic regression method, it is possible to talk about a DIF on that item (Swaminathan & Rogers, 1990).

Scope of this Study

This study epidemically benefited from differential item functioning (DIF) for the results of measurement in the research for bias. DIF means the differences depending on the below group's possibility of correct response for each skill of the psychological textures that would like to be measured with the substance (Lord, 1980; Camile, 1993; Embretson & Reise, 2000). On the concerning data, the research of DIF has been made, and the results also have been examined in each performance (Acar, 2008). The pivotal aim of the study is the comparison of the similarity between group coefficients and intercept that are gained from the analyses of logistic regression and generalized gradual linear models used in adjustment of DIF.

Method

Sample Characteristics

Population of the this study was 798307 students who took the 2006 student selection and placement tests for Secondary School Institutions Examination (SSIE) carried out by the Ministry of National Education in Turkey. Sampling of study covered stratified random sampling method, and data were gathered from 10727 students. Variables of the study were students' answers for Turkish test, their gender, and their socioeconomic status (SES). The three sample sizes of SES groups and gender groups are shown below in Table 1.

Table 1: Sample Size of the Three School District's SES Groups and Gender Groups

		Gender		Total (%)
		Male	Female	
Socio-economic statues	Lower	1241	1053	2294 (21%)
	Middle	2199	2512	4711 (44%)
	Upper	1864	1858	3722 (35%)
Total (%)		5304 (49%)	5423 (51%)	10727

A total of three pairs of group comparisons were made in the following manner:

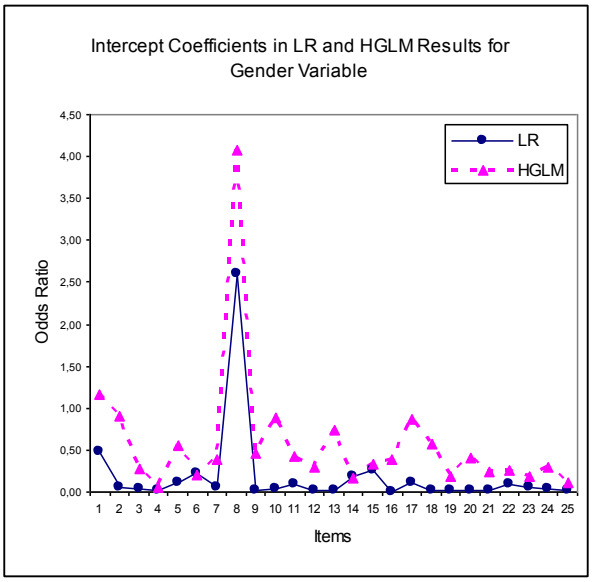
male-female (n = 10727), lower middle (n = 7005), and lower upper (n = 6016) SES groups were examined for item performance. Each pair of groups was analyzed through both the LR-DIF procedure and the HGLM-DIF model.

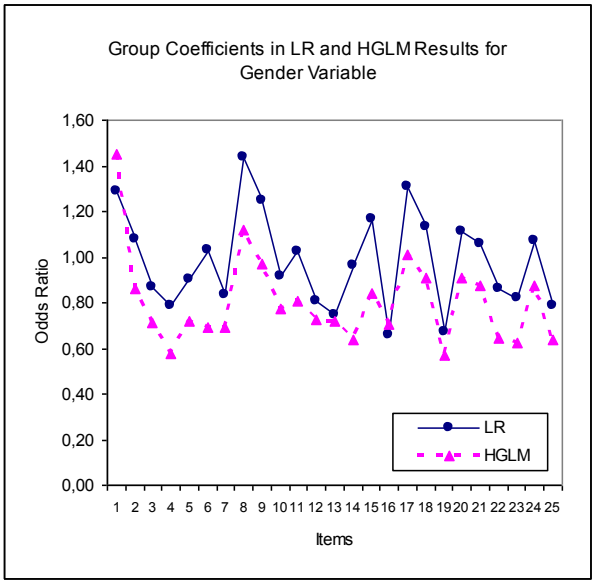
Data Analysis Procedure

The data used for this study consist of 25 items in the Turkish subtest items of the SSIE that applied in Turkey in 2006. The test items were coded “0” for incorrect and “1” for correct. The Turkish subtest items data were combined with gender and SES. Three pairs of comparison groups were used in three separate analyses, for which the reference group was coded as "0" and the focal group was coded as "1." The reference group was the *male* group in the gender group comparison, the *middle* group in the *lower middle*, and the *upper* group in the *lower upper* analyses.

Results and Comments

Results of the HGLM-DIF and LR-DIF analysis were presented in Appendix 1. The coefficients in the HGLM-DIF and the LR-DIF result are represented as odds ratio. There is a direct relationship between the coefficients produced by the odds ratios and the logit. For example, the interpretation of this odds ratio would be that the odds of a female examinee eliciting a correct answer are 1.45 times greater than that for a male examinee. Odds ratios are an important facet of HGLM and the LR methods on which it is based. HGLM takes the structure of the LR model and the interpretation of results based on it and adapts them to a situation where the data are hierarchical (Gaitanis, 2003). Figure 1 shows the line chart intercept and group coefficients odds ratio values of the HGLM-DIF and the LR-DIF.


Intercept Coefficients in LR and HGLM Results for Gender Variable
Odds Ratio
Items
LR
HGLM


Group Coefficients in LR and HGLM Results for Gender Variable
Odds Ratio
Items
LR
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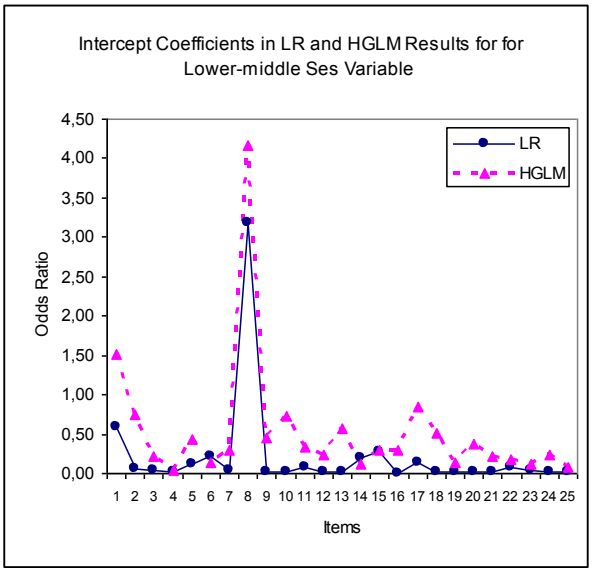
Intercept Coefficients in LR and HGLM Results for for Lower-middle Ses Variable
Odds Ratio
Items
LR
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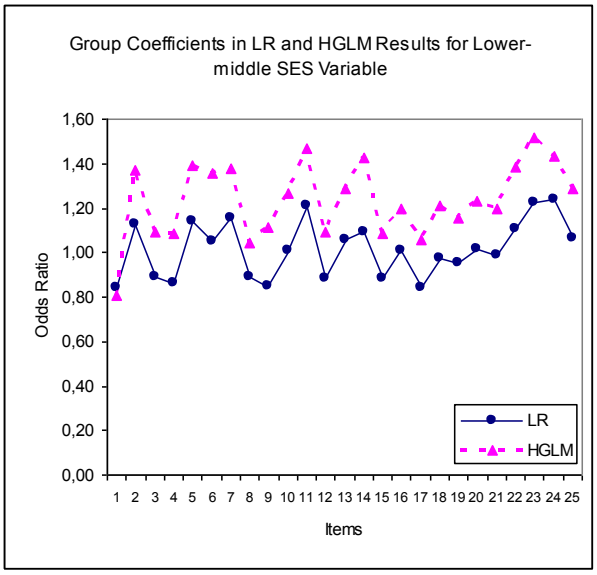
Group Coefficients in LR and HGLM Results for Lower-middle SES Variable
Odds Ratio
Items
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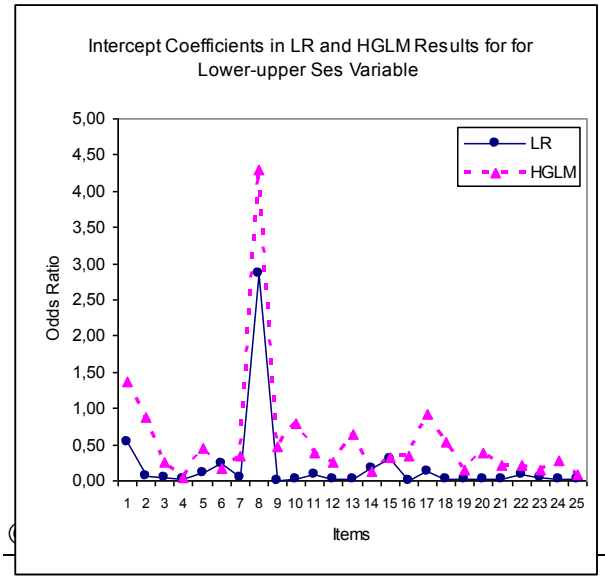
Intercept Coefficients in LR and HGLM Results for for Lower-upper Ses Variable
Odds Ratio
Items
LR
HGLM

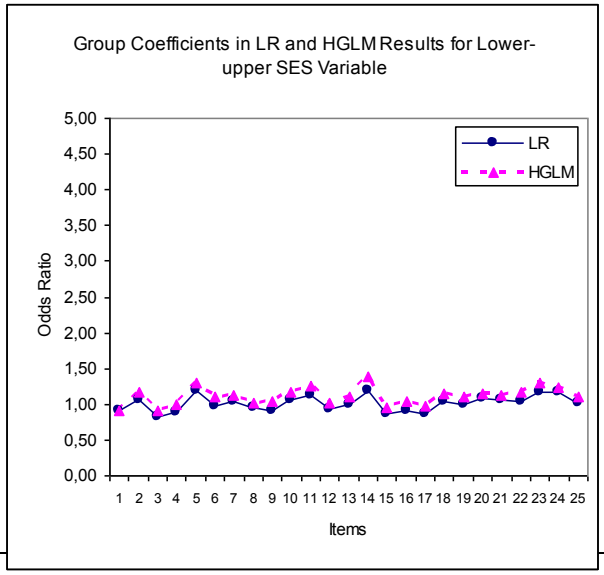
Group Coefficients in LR and HGLM Results for Lower-upper SES Variable
Odds Ratio
Items
LR
HGLM

Figure 1: Intercept and group coefficients odds ratio values of the HGLM and the LR

It can be explained that the intercept and group coefficents of the test items estimated by the HGLM-DIF and LR-DIFmethods that similar aptitude for 25 items. When the intercept coefficient of test items are assessed according to gender and SES parameters, it can be observed that odds ratio assessments gained from the HGLM-DIF methods are higher than odds ratio assessments gained from LR-DIF methods. However, when interaction coefficients of test items are according to the parameters of gender, the odds ratio assessments gained from LR-DIF methods are higher than odds ratio assessments gained from the HGLM-DIF methods. In the comparison for the SES parameter, odds ratio assessments gained from the HGLM-DIF methods are higher than odds ratio assessments gained from LR-DIF methods. When the relationship between odds ratio values of both methods are analyzed, the results that will be gained are in the Table 2.

Table 2: The correlation of group and intercept coefficients from LR-DIF and HGLM-DIF

	N	**Intercept coefficients**	**Interaction coefficients**
Gender Comparison (female-male)	10727	0.94	0.83
SES Comparison (lower-middle)	7005	0.95	0.94
SES Comparison (lower-upper)	6016	0.95	0.96

As shown in Table 2, according to the gender variable comparison, the correlation of intercept coefficients from the two methods is quite high (r = 0.94). The correlation of the group (interaction) coefficients show a higher correlation (r = 0.83), but the two coefficients are still high. According to lower-middle SES variable comparison, the correlation of intercept and interaction coefficients from the two methods also is quite high (r = 0.95 and r = 0.94). For lower-upper SES variable comparison, the correlation of intercept and interaction coefficients from the two methods is quite high (r = 0.95 and r = 0.96). On the other hand, HGLM procedure is equivalent to the LR method (Virginia, Howard & Robin, 2001). Chekki (2000) noted that each of the γ_{xx} (gamma) coefficients provides an interpretation. These γ_{xx} coefficients may be interpreted in the same way as logit coefficients in a logistic regression. Also, they may be converted into odds ratios by exponentiating them.

Conclusions and Recommendations

In this study, the similarities between the HGLM-DIF and the LR-DIF methods were discussed. The empirical investigation comparing the HGLM-DIF method with the LR-DIF method has provided evidence for the assertion that the HGLM-DIF procedure is equivalent to the LR-DIF method. The symmetry between these two methods has been identified in various aspects of the results, including correlation of group and intercept coefficients. The correlation of group and intercept coefficients from the two methods is quite perfect. HGLM-DIF may be strongly recommended for DIF analysis, if the outcome variable is binary, because the HGLM-DIF procedure does not require more effort and more time than LR-DIF procedure does.

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Appendix 1:

Gender N = 10727	LR-DIF		HGLM-DIF		Lower-middle N = 7005	LR-DIF		HGLM-DIF		Lower-upper N = 6016	LR-DIF		HGLM-DIF	
	Intercept Coefficient	Group Coefficient	Intercept Coefficient	Group Coefficient		Intercept Coefficient	Group Coefficient	Intercept Coefficient	Group Coefficient		Intercept Coefficient	Group Coefficient	Intercept Coefficient	Group Coefficient
1	0,478	1,290	1,158	1,450	**1**	0,586	0,840	1,512	0,809	**1**	0,528	0,919	1,358	0,899
2	0,064	1,080	0,913	0,860	**2**	0,059	1,129	0,751	1,368	**2**	0,071	1,063	0,874	1,175
3	0,039	0,868	0,275	0,713	**3**	0,036	0,888	0,215	1,092	**3**	0,043	0,822	0,256	0,919
4	0,019	0,787	0,050	0,576	**4**	0,019	0,860	0,036	1,084	**4**	0,018	0,889	0,040	1,003
5	0,115	0,905	0,554	0,716	**5**	0,114	1,138	0,427	1,389	**5**	0,105	1,181	0,459	1,294
6	0,215	1,029	0,199	0,695	**6**	0,209	1,050	0,145	1,355	**6**	0,226	0,978	0,178	1,112
7	0,053	0,835	0,392	0,694	**7**	0,046	1,155	0,285	1,380	**7**	0,050	1,033	0,349	1,131
8	2,602	1,435	4,068	1,116	**8**	3,174	0,890	4,164	1,044	**8**	2,872	0,943	4,291	1,010
9	0,010	1,248	0,470	0,971	**9**	0,011	0,849	0,444	1,113	**9**	0,010	0,904	0,476	1,041
10	0,028	0,912	0,882	0,775	**10**	0,028	1,010	0,718	1,269	**10**	0,024	1,058	0,780	1,169
11	0,088	1,026	0,427	0,807	**11**	0,079	1,211	0,334	1,466	**11**	0,090	1,135	0,394	1,245
12	0,017	0,804	0,287	0,725	**12**	0,017	0,885	0,235	1,092	**12**	0,018	0,921	0,253	1,019
13	0,014	0,749	0,733	0,720	**13**	0,011	1,058	0,562	1,286	**13**	0,014	0,987	0,650	1,113
14	0,179	0,964	0,161	0,639	**14**	0,190	1,091	0,118	1,424	**14**	0,164	1,182	0,122	1,389
15	0,256	1,165	0,333	0,840	**15**	0,278	0,885	0,287	1,088	**15**	0,297	0,871	0,328	0,957
16	0,007	0,658	0,387	0,703	**16**	0,006	1,008	0,298	1,197	**16**	0,007	0,919	0,346	1,034
17	0,115	1,307	0,869	1,011	**17**	0,139	0,840	0,839	1,061	**17**	0,128	0,860	0,911	0,977
18	0,023	1,132	0,567	0,908	**18**	0,025	0,972	0,509	1,209	**18**	0,026	1,038	0,542	1,136
19	0,024	0,670	0,187	0,569	**19**	0,021	0,953	0,134	1,157	**19**	0,022	1,006	0,142	1,098
20	0,021	1,109	0,409	0,908	**20**	0,022	1,019	0,367	1,230	**20**	0,023	1,078	0,391	1,157
21	0,019	1,057	0,235	0,874	**21**	0,022	0,991	0,209	1,193	**21**	0,018	1,059	0,221	1,135
22	0,090	0,860	0,262	0,642	**22**	0,085	1,109	0,184	1,385	**22**	0,078	1,043	0,221	1,160
23	0,054	0,823	0,180	0,622	**23**	0,044	1,226	0,123	1,520	**23**	0,051	1,168	0,144	1,304
24	0,028	1,071	0,292	0,874	**24**	0,025	1,235	0,241	1,430	**24**	0,031	1,160	0,280	1,237
25	0,016	0,786	0,103	0,635	**25**	0,014	1,068	0,074	1,289	**25**	0,016	1,009	0,087	1,102